# Johnson’s <br> TABLES 

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## Johnson's Tables.

## STADIA AND EARTH-WORK TABLES.

Four-place Logarithms, Logarithmic Traverse
Table, Natural Functions, Map
Projections, etc., etc.
THEORY AND $\stackrel{\text { Reprinted from }}{\text { PRACTICE }}$ OF SURUEYING.

BY<br>J. B. JOHNSON, professor of civil enginerring, washington university, st. louis

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## NOTE BY THE AUTHOR.

THE great use made by engineers of three of the following tables, viz., the Four-place Logarithmic Table, the Stadia Table, and the table giving Prismoidal Volumes, has necessitated the binding of these in more convenient form than that in which they first appeared in the Theory and Practice of Surveying. Since the cost is not materially increased by additional pages, the remaining tables are also included, as well as the entire chapter on the Measurement of Volumes.

The Stadia Tables were computed by Mr. Arthur Winslow, State Geologist of Missouri, and first published by the Pennsylvania Geological Survey. The four-place logarithm tables were originally taken from Lee's Tables and Formulæ, a publication of the U. S. Engineer Corps. The table giving Volumes by the Prismoidal Formula was computed by the Author, It is the only table, he believes, giving volumes by the prismoidal formula at one operation. It may also be used for Mean End-areas. Tables IV and VIII are also original in their arrangement.
J. B. J.

## EXPLANATION OF TABLES.

Tables I, II, III, VI, and VII require no explanation.
Table IV gives logarithmic sines and cosines to four places for computing latitudes and departures when the angles are read from zero to 360 degrees. It can of course be used for bearings reading from zero to 90 degrees, as is ordinarily done in compass work. In stadia work, and always in transit work where the instrument is graduated continuously to 360 degrees, this table will be found very convenient for coördinating traverse lines, as well as for computing latitudes and departures for closed surveys.

From zero to 5 degrees, and from 85 to 90 degrees, the tables give values for each minute of arc without tabular differences. From 5 to 45 degrees values are given for each io minutes of arc with tabular differences for the log. sines, and from 45 to 85 degrees with tabular differences for the io-minute increments for the log. cosines. In the other cases the tabular difference is so small as to be readily taken at sight. Table $\mathrm{III}_{\mathrm{A}}$ can of course be used in place of Table IV if preferred.

Table V gives horizontal distance and difference of elevation for inclined sights in stadia work. The true equations of reduction are:

$$
\text { Hor. Dist. }=r \cos ^{2} v+(c+f) \cos v, ~ . ~ . ~ . ~(1) ~
$$

and

$$
\begin{equation*}
\text { Dif. Elev. }=r \cos v \sin v+(c+f) \sin v ; . \tag{2}
\end{equation*}
$$

where
$r=$ reading of distance on stadia rod when held vertically;
$v=$ vertical angle with the horizon;
$f=$ focal length of objective ;
$c=$ distance from objective to centre of instrument.
The tables give the values for the first term only of the second member. The values for the second term are given at the bottom of the page, the constant term $(c+f)$ in the above equations being there called " $c$." The sum of these two distances, viz., distance from centre of instrument to objective plus distance from cross-wires to objective, varies in different instruments from nine to fifteen inches. Three values of this second term are given, therefore, one corresponding to $c+f=$ 0.75 foot, one to $c+f=1.00$ foot, and one to $c+f=1.25$ foot. In ordinary work these corrections may be neglected. See chapter on Stadia Surveying in the Theory and Practice of Surveying.

A Reduction Diagram, printed from an engraved plate 20 by 24 inches, has been prepared with great care, giving corrections to the horizontal distance read, and the differences of elevation, for inclined sights, as shown by the table, not including the $(c+f)$ term. For all angles below $6^{\circ}$ and distances less than 1500 feet, with differences of elevation less than 50 feet, this diagram is much preferable to the table. The results are found at one operation, to the nearest tenth of a foot, with great rapidity. It can be procured from the publisher of these tables, printed on heavy lithographic paper, price 50 cents, post paid.

Table VIII gives the coördinates to be used in the polyconic projection of maps. It is fully explained in the chapter on Projection of Maps in the Surveying.

Tables IX and X will be found very useful in sewer and hydraulic work where Kutter's formula is to be used. They
are fully explained in the chapter on Hydrographic Surveying.

Table XI gives correct volumes of prismoids, by the prismoidal formula.

For the benefit of railroad engineers and others who either do not possess a copy of the Surveying, or who do not have it by them, the entire chapter on the Measurement of Volumes is here inserted. At least seven pages of this chapter is requisite to a full explanation of the table, and for the sake of completeness, and to show the superiority of this table over any table of volumes from mean end-areas, or by the use of diagonals, it has been thought best to insert the entire chapter.

Table XII gives the azimuth of Polaris at any hour-angle. By its use an observation for azimuth to the nearest minute of arc can be made at any hour when the star is visible, provided the local time is known to within one or two minutes. When the observation is taken two hours from the time of elongation, the local time need not be known nearer than five minutes. A detailed explanation of its use is given in the Surveying, Art. $38 \mathrm{I}_{\mathrm{A}}$.

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## CHAPTER XIII.

## THE MEASUREMENT OF VOLUMES.

310. Proposition.-The volume of any doubly-truncated prism or cylinder, bounded by plane ends, is equal to the area of a right section into the length of the element through the centres of gravity of the bases, or it is equal to the area of either base into the altitude of the element joining the centres of gravity of the bases, measured perpendicular to that base.

Let $A B C D$, Fig. Io7, be a cylinder, cut by the planes $O C$ and $O B$, the unsymmetrical right section $E F$ being shown in plan in $E^{\prime} F^{\prime}$. Whatever position the cutting planes may have, if they are not parallel they will intersect in a line. This line of intersection may be taken perpendicular to the paper, and the body would then appear as shown in the figure, the line of intersection of the cutting planes being projected at $O$.

$$
\begin{aligned}
\text { Let } A & =\text { area of the right section; } \\
\Delta A & =\text { any very small portion of this area; } \\
x & =\text { distance of any element from } O ; \\
\text { then } a x & =\text { height of any element at a distance } x \text { from } O .
\end{aligned}
$$

An elementary volume would then be $a x \Delta A$, and the total volume of the solid would be $\Sigma a x \Delta A$.

Again, the total volume is equal to the mean or average height of all the elementary volumes multiplied by the area of the right section.

The mean height of the elementary volumes is, therefore,
$\frac{\Sigma a x \Delta A}{A}=\frac{a \Sigma x \Delta A}{A}$. But $\frac{\Sigma x \Delta A}{A}$ is the distance from $O$ to the centre of gravity, $G$, of the right section,* and $a$ times this dis tance is the height of the element $L K$ through this point. Therefore, the mean height is the height through the centre of


Fig. 107.
gravity of the base, and this into the area of the right section is the volume of the truncated prism or cylinder. The truth of the alternative proposition can now readily be shown.

Corollary. When the cylinder or prism has a symmetrical cross-section, the centre of gravity of the base is at the centre of the figure, and the length of the line joining these centres is the mean of any number of symmetrically chosen exterior elements. For instance, if the right section of the prism be a regular polygon, the height of the centre element is the mean of the length of all the edges. This also holds true for parallelograms, and hence for rectangles. Here the centres of gravity

[^0]of the bases lie at the intersections of the diagonals; and since these bisect each other, the length of the line joining the intersections is the mean of the lengths of the four edges. The same is true of triangular cross-sections.

3II. Grading over Extended Surfaces.-Lay out the area in equal rectangles of such a size that the surfaces of the several rectangles may be considered planes. For common rolling ground these rectangles should not be over fifty feet on a side. Let Fig. io8 represent such an area. Drive pegs at

the corners, and find the elevation of the ground at each intersection by means of a level, reading to the nearest tenth of a foot, and referring the elevations to some datum-plane below the surface after it is graded. When the grading is completed, relocate the intersections from witness-points that were placed outside the limits of grading, and again find the elevations at these points. The several differences are the depths of excavation (or fill) at the corresponding corners. The contents of any partial volume is the mean of the four corner heights into the area of its cross-section. But since the rectangular areas were made equal, and since each corner height will be used as many times as there are rectangles joining at that corner, we have, in cubic yards,

$$
\begin{equation*}
V=\frac{A}{4 \times 2 \eta}\left[\Sigma h_{1}+2 \Sigma h_{2}+3 \Sigma h_{3}+4 \Sigma h_{1}\right] . \tag{1}
\end{equation*}
$$

The subscripts denote the number of adjoining rectangles the area of each of which is $A$.

From this equation we may frame a
Rule.-Take each corner height as many times as there are partial areas adjoining it, add them all together, and multiply by one fourth of the area of a single rectangle. Tnis gives the volume in cubic feet. To obtain it in cubic yards, divide by twenty-seven.

If the ground be laid out in rectangies, 30 feet by 36 feet, then $\frac{A}{4 \times 27}=\frac{1080}{108}=10 ;$ and if the elevations be taken to the nearest tenth of a foot, then the sum of the multiplied corner heights, with the decimal point omitted, is at once the the amount of earthwork in cubic yards. This is a common way of doing this work. In borrow-pits, for which this method is peculiarly fitted, the elementary areas would usually be smaller.

In general, on rolling ground, a plane cannot be passed through the four corner heights. We may, however, pass a plane through any three points, and so with four given points

on a surface either diagonal may be drawn, which with the bounding lines makes two surfaces. If the ground is quite irregular, or if the rectangles are taken pretty large, the surveyor may note on the ground which diagonal would most
nearly fit the surface. Let these be sketched in as shown in Fig. 109. Each rectangular area then becomes two triangles, and when computed as triangular prisms, each corner height at the end of a diagonal is used twice, while the two other corner heights are used but once. That is, twice as much weight is given to the corner heights on the diagonals as to the others. In Fig. 109, the same area as that in Fig. 108 is


Fig.ifo. shown with the diagonals drawn which best fit the surface of the ground. The numbers at the corners indicate how many times each height is to be used. It will be seen that each height is used as many times as there are triangles meeting at that corner. To derive the formula for this case, take a single rectangle, as in Fig. 110, with the diagonal joining corners 2 and 4 . Let $A$ be the area of the rectangle. Then from the corollary, p. 395, we have for the volume of the rectangular prism, in cubic yards,

$$
\begin{align*}
V & =\frac{A}{2 \times 27}\left(\frac{h_{1}+h_{2}+h_{4}}{3}+\frac{h_{2}+h_{3}+h_{4}}{3}\right) \\
& =\frac{A}{6 \times 27}\left(h_{1}+2 h_{2}+h_{3}+2 h_{4}\right) \ldots . \tag{2}
\end{align*}
$$

For an assemblage of such rectangular prisms as shown in Fig. Iog, the diagonals being drawn, we have, in cubic yards,

$$
\begin{array}{r}
V=\frac{A}{6 \times 27}\left[\Sigma h_{1}+2 \Sigma h_{\mathrm{a}}+3 \Sigma h_{\mathrm{3}}+4 \Sigma h_{4}+5 \Sigma h_{\mathrm{b}}\right. \\
\left.+6 \Sigma h_{\mathrm{6}}+7 \Sigma h_{7}+8 \Sigma h_{\mathrm{B}}\right] ; \tag{3}
\end{array}
$$

where $A$ is the area of one rectangle, and the subscripts denote the number of triangles meeting at a corner.

As a check on the numbering of the corners, Fig. 109, add them all together and divide by six. The result should be the number of rectangles in the figure. In this case, if the rectangles be taken 36 feet by 45 feet, or, better, 40 feet by 40.5 . feet, then the sum of the multiplied heights with the decimal point omitted is the number of cubic yards of earthwork, the corner heights having been taken out to tenths of a foot.

The method by diagonals is more accurate than that by rectangles simply, the dimensions being the same; or, for equal degrees of exactness larger rectangles may be used with diagonals than without them, and hence the work materially reduced. In any case some degree of approximation is necessary.
312. Approximate Estimates by means of Contours.(A) Whenever an extended surface of irregular outline is to be graded down, or filled up to a given plane (not a warped or curved surface), a near approximation to the amount of cut or fill may be made from the contour lines. In Fig. III the full curved lines are contours, showing the original surface of the ground. Every fifth one is numbered, and these were the contours shown on the original plat. Intermediate contours one foot apart have been interpolated for the purpose of making this estimate. The figures around the outside of the bounding lines give the elevations of those points after it is graded down. The straight lines join points of equal elevation after grading; and since this surface is to be a plane these lines are surface or contour lines after grading. Wherever these two sets of contour lines intersect, the difference of their elevations is the depth of cut or fill at that point. If now we join the points of equal cut or fill (in this case it is all in cut), we obtain a new set of curves, shown in the figure by dotted lines, which may be used for estimating the amount of earthwork. The dotted boundaries are the horizontal projections of the traces on the natural surface of planes parallel to the final
graded surface which are uniformly spaced one foot apart vertically. These projected areas are measured by the planimeter and called $A_{1}, A_{2}, A_{3}$, etc. Each area is bounded by the dotted line and the bounding lines of the figure, since on these


Fig. iII.
bounding lines all the projections of all the traces unite, the slope here being vertical. For any two adjoining layers we have, by the prismoidal formula* as well as by Simpson's onethird rule,

$$
\begin{equation*}
V_{x-3}=\frac{h}{3}\left(A_{1}+4 A_{2}+A_{3}\right), \tag{I}
\end{equation*}
$$

where $h$ is the common vertical distance between the projected areas.

[^1]For the next two layers we would have, similarly,

$$
\begin{equation*}
V_{3-5}=\frac{h}{3}\left(A_{5}+4 A_{4} A_{5}\right) ; . \tag{2}
\end{equation*}
$$

or for any even number of layers we would have, in cubic yards,

$$
V=\frac{h}{3 \times 27}\left(A_{1}+4 A_{3}+2 A_{3}+4 A_{4}+2 A_{6}+\ldots A_{n}\right),(3)
$$

where $n$ is an odd number, $h$ and $A$ being in feet and square feet respectively.
$(B)$ Whenever the final surface is not to be a plane, but warped, undulating, or built to regular outlines like a fortification, a reservoir embankment, or terraced grounds, a different method should be employed.

In the former method the areas bounded by the dotted lines were areas cut out by planes parallel to the final plane surface, passed one foot apart vertically. But since the map shows only the horizontal projections of these planes, these projections, multiplied by the vertical distance between them, would give the true volumes.

When the final surface is not to be a plane, proceed as follows: First make a careful contour map of the ground. Then lay down on this map a system of contour lines, corresponding in elevation to the first set of contours, but in a different colored ink, which will accurately represent the final surface desired. This second set of contours would be a series of straight lines if a regular surface, composed of plane faces, was to be constructed, but would be curving lines if the ground were to be brought to a final curving or undulating surface.

The closed figures bounded by the two sets of intersecting contours of the same elevation are horizontal areas of cut or fill, separated by the common vertical distance between
contours. The volumes here defined are oblique solids bounded by horizontal planes at top and bottom, and are a species of prismoid. The volume of one of these prismoids is found by applying the prismoidal formula to it, finding the end areas by means of a planimeter, and taking the length as the

vertical distance between contours. If the contours be drawn close enough together, then each alternate contour-area may be used as a middle area, and the length of the prismoid taken at twice the vertical distance between contours; or the volume
may be computed by either of the formulas (12), (13), (I4), or ( 15 ) of Appendix C, where the $h$ 's would here become the end areas and $l$ the vertical distance between contours.

Example: Let it be required to build a square reservoir on a hillside, which shall be partly in excavation and partly in embankment, the ground being such as shown by the full contour lines in Fig. III $\alpha$.*

The contours, for the sake of simplicity and brevity, are spaced five feet apart. The top of the wall, shown by the full lines making the square, is io feet wide and at an elevation of 660 feet. The reservoir is 20 feet deep, with side slopes, both inside and outside, of two to one, making the bottom elevation 640 feet, and 20 feet square, the top being iCO feet square on the inside. The dotted lines are contours of the finished slopes, both inside and out, at elevations shown on the figure. The areas in fill all fall within the broken line marked $a b c d e$ $f g h i k$, and the cut areas all fall within the broken line marked $a b c d$ ef $g o$. These broken lines are grade lines. The horizontal sectional areas in fill and cut are readily traced by following the closed figures formed by contours of equal elevation, thus-

| At 640 foot level sectional area in fill is pst. |
| :--- |
| " 650 " " |
| " 650 " |

The other areas are as easily traced. In the figure the lines have all been drawn in black. In practice they should be drawn in different colors to avoid confusion.

This second method should be used in all cases where the graded area is considerable and the final relief form is not a plane. If the contours be carefully determined and be taken

[^2]near enough together, the method will give as accurate results as may be obtained in any other way. The volume may be computed by eq. (3) of this article, where the areas are the horizontal sectional areas bounded by contours of equal elevation, and $h$ is the vertical distance between contours.

When these methods are used for final estimates, the contours should be carefully determined, and spaced not more than two feet apart on steep slopes and one foot apart on low slopes.
313. The Prismoid is a solid having parallel end areas, and may be composed of any combination of prisms, cylinders, wedges, pyramids, or cones or frustums of the same, whose bases and apices lie in the end areas. It may otherwise be defined as a volume generated by a right-line generatrix moving on the bounding lines of two closed figures of any shapes which lie in parallel planes as directrices, the generatrix not necessarily moving parallel to a plane director. Such a solid would usually be bounded by a warped surface, but it can always be subdivided into one or more of the simple solids named above.

Inasmuch as cylinders and cones are but special forms of prisms and pyramids, and warped surface solids may be divided into elementary forms of them, and since frustums may also be subdivided into the elementary forms, it is sufficient to say that all primoids may be decomposed into prisms, wedges, and pyramids. If a formula can be found which is equally applicable to all of these forms, then it will apply to any combination of them. Such a formula is called
314. The Prismoidal Formula.

Let $A=$ area of the base of a prism, wedge, or pyramid;
$A_{1} A_{m}, A_{2}=$ the end and middle areas of a prismoid, or of any of its elementary solids;
$h=$ altitude of the prismoid or elementary solid.

Then we have, For Prisms,

$$
V=h A=\frac{h}{6}\left(A_{1}+4 A_{m}+A_{2}\right) \ldots . .(\mathrm{I})
$$

For Wedges,

$$
\begin{equation*}
V=\frac{\hbar A}{2}=\frac{\hbar}{6}\left(A_{1}+4 A_{m}+A_{2}\right) \ldots \tag{2}
\end{equation*}
$$

For Pyramids,

$$
\begin{equation*}
V=\frac{h A}{3}=\frac{h}{6}\left(A_{1}+4 A_{m}+A_{2}\right) \ldots . \tag{3}
\end{equation*}
$$

Whence for any combination of these, having all the common altitude $h$, we have

$$
\begin{equation*}
V=\frac{\hbar}{6}\left(A_{1}+4 A_{m}+A_{2}\right), \cdots \cdots \tag{4}
\end{equation*}
$$

which is the prismoidal formula.
It will be noted that this is a rigid formula for all prismoids. The only approximation involved in its use is in the assumption that the given solid may be generated by a right line moving over the boundaries of the end areas.

This formula is used for computing earthwork in cuts and fills for railroads, streets, highways, canals, ditches, trenches, levees, etc. In all such cases, the shape of the figure above the natural surface in the case of a fill, or below the natural surface in the case of a cut, is previously fixed upon, and to complete the closed figure of the several cross-section areas only the outline of the natural surface of the ground at the section remains to be found. These sections should be located so near together that the intervening solid may fairly be as
sumed to be a prismoid. They are usually spaced 100 feet apart, and then intermediate sections taken if the irregularities seem to require it.

The area of the middle section is never the mean of the two end areas if the prismoid contains any pyramids or cones among its elementary forms. When the three sections are similar in form, the dimensions of the middle area are always the means of the corresponding end dimensions. This fact often enables the dimensions, and hence the area of the middle section, to be computed from the end areas. Where this cannot be done, the middle section must be measured on the ground, or else each alternate section, where they are equally spaced, is taken as a middle section, and the length of the prismoid taken as twice the distance between cross-sections. For a continuous line of earthwork, we would then have, in cubic yards,

$$
\begin{equation*}
V=\frac{l}{3 \times 27}\left(A_{1}+4 A_{8}+2 A_{3}+4 A_{4}+2 A_{6}+4 A_{6} \ldots+A_{n}\right), \tag{I}
\end{equation*}
$$

where $l$ is the distance between sections in feet. This is the same as equation (3), p. 40 I . Here the assumption is made that the volume lying between alternate sections conforms sufficiently near to the prismoidal forms.
315. Areas of Cross-sections. - In most cases, in practice at least, three sides of a cross-section are fixed by the conditions of the problem. These are the side slopes in both cuts and fills, the bottom in cuts and the top in embankments, or fills. It then remains simply to find where the side slopes will cut the natural surface, and also the form of the surface line on the given section. Inasmuch as stakes are usually set at the points where the side slopes cut the surface, whether in cut or fill, such stakes are called slope-stakes, and they are set at the time
the cross-section is taken. The side slopes are defined as so much horizontal to one vertical. Thus a slope of $\mathrm{I} \frac{1}{2}$ to I means that the horizontal component of a given portion of a slopeline is $\mathrm{I} \frac{1}{2}$ times its vertical component, the horizontal component always being named first. The slope-ratio is the ratio of the horizontal to the vertical component, and is therefore always the same as the first number in the slope-definition. Thus for a slope of $\mathrm{I}_{\frac{1}{2}}$ to I the slope-ratio is $\mathrm{I}_{\frac{1}{2}}$.
316. The Centre and Side Heights.-The centre heights are found from the profile of the surface along the centre line, on which has been drawn the grade line of the proposed work. These are carefully drawn on cross-section paper, when the height of grade at each station above or below the surface line can be taken off. These centre heights, together with the width of base and side slopes in cuts and in fills, are the necessary data for fixing the position of the slope-stakes. When these are set for any section as many poirts on the surface line joining them may be taken as desired. In ordinary rolling ground usually no intermediate points are taken, the centre point being already determined. In this case three points in the surface line are known, both as to their distance out from the centre line and as to their height above the grade line. Such sections are called "three-level sections," the surface lines being assumed straight from the slope-stakes to the centre stake.

## 317. The Area of a Three-level Section.

Let $d$ and $d^{\prime}$ be the distances out, and
$h$ and $h^{\prime}$ the heights above grade of right and left slopestakes, respectively;
$D$ the sum of $d$ and $d^{\prime}$,
$c$ the centre height,
$r$ the slope-ratio,
$w$ the width of bed.

Then the area $A B C D E$ is equal to the sum of the four trian. gles $A E w, B C w, w C D$, and $w E D$. Or,

$$
\begin{equation*}
A=\frac{\left(d+d^{\prime}\right) c+\left(h+h^{\prime}\right) \frac{w}{2}}{2} . \tag{I}
\end{equation*}
$$

This area is also equal to the sum of the triangles $F C D$ and $F E D$, minus the triangle $A F B$. Or,

$$
\begin{equation*}
A=\left(c+\frac{w}{2 r}\right) \frac{D}{2}-\frac{w^{2}}{4 r} . . . . \tag{2}
\end{equation*}
$$



Fig. 112.
Equation (2) can also be obtained directly from equation (I) by substituting for $h$ and $h^{\prime}$ in (I) their values in terms of $d$ and $w, h=\frac{d-\frac{w}{2}}{r}$, and then putting $D=d+d^{\prime}$. Equation
(2) has but two variables, $c$ and $D$, and is the most convenient one to use.
318. Cross-sectioning.-It will be seen from Fig. iI2 that in the case of a three-level section the only quantities to be determined in the field are the heights, $h$ and $h^{\prime}$, and the distances out, $d$ and $d^{\prime}$, of the slope-stakes. These are found by trial. A levelling instrument is set up so as to read on the
three points $C, D, E$, and the rod held first at $D$. The reading here gives the height of instrument above this point. Add this algebraically to the centre height (which may be negative, and which has been obtained from the profile for each station), and the sum is the height of instrument above (or below) the grade line. If the ground were level transversely, the distance out to the slope-stakes would be

$$
d=c r+\frac{w}{2} .
$$

But this is not usually the case, and hence the distance out must be found by trial. If the ground slopes $\left\{\begin{array}{c}\text { down } \\ \text { up }\end{array}\right\}$ from the centre line in a $\left\{\begin{array}{l}\text { fill } \\ \text { cut }\end{array}\right\}$ the distance out will evidently be more than that given by the above equation, and vice versa. The rodman estimates this distance, and holds his rod at a certain measured distance out, $d_{1}$. The observer reads the rod, and deducts the reading from the height of instrument above grade (or adds it to the depth of instrument below grade), and this gives the height of that point, $h_{1}$, above or below grade. Its distance out, then, should be $d=h_{1} r+\frac{w}{2}$. If this be more than the actual distance out, $d_{1}$, the rod is set farther out; if less, it is moved in. The whole operation is a very simple one in practice, and the rodman soon becomes very expert in estimating nearly the proper position the first time.

In heavy work-that is, for large cuts or fills, and for irregular ground-it may be necessary to take the elevation and distance out of other points on the section in order to better determine its area. These are taken by simply reading on the rod at the critical points in the outline, and measuring the distances out from the centre. The points can then be plotted
on cross-section paper and joined by straight or by free-hand curved lines. In the latter case the area should be determined by planimeter.
319. Three-level Sections, the Upper Surface consisting of two Warped Surfaces.-If the three longitudinal lines joining the centre and side heights on two adjacent threelevel sections be used as directrices, and two generatrices, one on each side the centre, be moved parallel to the end areas as plane directers, two warped surfaces are generated, every crosssection of which parallel to the end areas is a three-level section. These same surfaces could be generated by two longitudinal generatrices, moving over the surface end-area lines as directrices. The surface would therefore be a prismoid, and its exact volume would be given by the prismoidal formula. The middle area in this case is readily found, since the center and side heights are the means of the corresponding end dimensions.

The prismoidal formula, giving volumes in cubic yards,

$$
\begin{equation*}
V=\frac{l}{6 \times 27}\left(A_{1}+4 A_{m}+A_{2}\right), \quad \cdots \cdots \tag{I}
\end{equation*}
$$

could therefore be written

$$
\begin{align*}
V=\frac{l}{12 \times 27}\left[\left(c_{1}\right.\right. & \left.+\frac{z v}{2 r}\right) D_{1}+\left(c_{2}+\frac{w}{2 r}\right) D_{2} \\
& \left.+4\left(c_{m}+\frac{w}{2 r}\right) D_{m}\right]-\frac{l w w^{2}}{4 \times 27 r} . \tag{2}
\end{align*}
$$

This equation is derived directly from eq. (I) above, and eq. (2), p. 406. The quantity $\frac{w}{2 r}$ is the distance from the grade-plane
to the intersection of the side slopes, and is a constant for any given piece of road. It would have different values, however, in cuts and fills on the same line.

For brevity, let

$$
\frac{w}{2 r}=c_{0} ; \quad \text { and } \quad \frac{l w w^{2}}{4 \times 27 r}=\frac{l w c_{0}}{54}=K
$$

Here $K$ is the volume of the prism of earth, 100 feet long, included between the roadbed and side slopes. It is first included in the computation and then deducted. It is also a constant for a given piece of road.

Equation (2) now becomes

$$
V=\frac{l}{12 \times 2 \eta}\left[\left(c_{1}+c_{0}\right) D_{1}+\left(c_{2}+c_{0}\right) D_{2}+4\left(c_{m}+c_{0}\right) D_{m}\right]-K, .(3)
$$

where $c_{m}$ and $D_{m}$ are the means of $c_{1} c_{2}$ and $D_{1} D_{2}$, respectively.
This equation involves but two kinds of variables, $c$ and $D$, and is well adapted to arithmetical, tabular, or graphical computation. Thus if $l=100 ; w=18$; and $r=1 \frac{1}{2}$; then $c_{0}=6$; and $K=200$; and equation (3) becomes

$$
\begin{equation*}
V=\frac{100}{324}\left[\left(c_{1}+6\right) D_{1}+\left(c_{2}+6\right) D_{2}+4\left(c_{m}+6\right) D_{m}\right]-200 \tag{4}
\end{equation*}
$$

If the total centre heights (to intersection of side slopes) be represented by $C_{1}, C_{2}$, and $C_{m}$, then eq. (3) becomes, in general,

$$
\begin{equation*}
V=K^{\prime}\left(C_{1} D_{1}+C_{2} D_{2}+4 C_{m} D_{m}\right)-K, . \tag{5}
\end{equation*}
$$

where $K^{\prime}=\frac{100}{324}$, and is independent of width of bed and of slopes.

For any given piece of road, the constants $K, K^{\prime}$, and $c_{0}$ are known, and for each prismoid the $C$ 's and $D$ 's are observed, hence for any prismoid all the quantities in eq. (5) are known.
320. Construction of Tables for Prismoidal Computa-tion.-If a table were prepared giving the products $K^{\prime} C D$ for various values of $C$ and $D$, it could be used for evaluating equation (3), which is the same as equation (5). The arguments would be the total widths $\left(D_{1}\right)$, and the centre heights $\left(C_{1}\right)$. Such a table would have to be entered three times for each prismoid, first with $C_{1}$ and $D_{1}$; second with $C_{2}$ and $D_{2}$; and finally with $C_{m}$ and $D_{m}$. If four times the last tabular value be added to the sum of the other two, and $K$ subtracted, the result is the true volume of the prismoid.
values of $c_{o}\left(=\frac{z v}{2 r}\right)$ And $K\left(=\frac{l z v^{2}}{4 \times 2 \eta r}\right)$ FOR various widths AND SLOPES.

| Width Roadbed. | Stopes. |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2/2 to 1. |  | 2/2 to 1. |  | 3/4 to 1. |  | 1 to 1. |  | 11/2 to 1. |  | $11 / 2$ to 1. |  | $13 / 4$ to 1. |  | 2 to 1. |  |
|  | $C_{0}$ | $K$ | $C_{0}$ | $K$ | $C_{0}$ | $K$ | $c_{0}$ | $K$ | $C_{0}$ | $K$ | $C_{0}$ | $K$ | $C_{0}$ | $K$ | $C_{0}$ | $K$ |
| 10 | 20 | 370 | 10 | 185 | 6.7 | 123 | 5.0 | 93 | 4.0 | 74 | $3 \cdot 3$ | 62 | 2.9 | 53 | 2.5 | 46 |
| 11 | 22 | 448 | II | 224 | $7 \cdot 3$ | 149 | 5.5 | 112 | 4.4 | 90 | 3.7 | 75 | 3.1 | 64 | 2.8 | 56 |
| 12 | 24 | 533 | 12 | 266 | 8.0 | 178 | 6.0 | 133 | 4.8 | 107 | 4.0 | 89 | $3 \cdot 4$ | 76 | 3.0 | 67 |
| 13 | 26 | 626 | 13 | 313 | 8.7 | 209 | 6.5 | 157 | 5.2 | 125 | $4 \cdot 3$ | 104 | $3 \cdot 7$ | 89 | 3.2 | 78 |
| 14 | 28 | 725 | 14 | ${ }_{3} 6$ | $9 \cdot 3$ | 242 | 7.0 | 181 | 5.6 | 145 | 4.7 | 121 | 4.0 | 104 | 3.5 | 9 F |
| 15 | 30 | 833 | 15 | 417 | 10.0 | 278 | 7.5 | 208 | 6.0 | 167 | 5.0 | I39 | $4 \cdot 3$ | 119 | 3.8 | 104 |
| 16 | 32 | 948 | 16 | 474 | 10.7 | 316 | 8.0 | 237 | 6.4 | 190 | $5 \cdot 3$ | 158 | 4.6 | 135 | 4.0 | 118 |
| 17 | 34 | 1070 | 17 | 535 | II. 3 | 357 | 8.5 | 268 | 6.8 | ${ }^{214}$ | 5.7 | 178 | 4.9 | 153 | 4.2 | 134 |
| 18 | 36 | 1200 | 18 | 600 | 12.0 | 400 | 9.0 | 300 | 7.2 | 240 | 6.0 | 200 | 5.1 | 171 | 4.5 | 150 |
| 19 | 38 | 1337 | 19 | 668 | 12.7 | 446 | 9.5 | 334 | 7.6 | 267 | 6.3 | 223 | $4 \cdot 4$ | 191 | 4.8 | 167 |
| 20 | 40 | I48I | 20 | 740 | 13.3 | 494 | 10.0 | 370 | 8.0 | 296 | 6.7 | 247 | 5.7 | 212 | 5.0 | 185 |
| 21 | 42 | 1633 | 21 | 816 | 14.0 | 544 | 10.5 | 408 | 8.4 | 327 | 7.0 | 272 | 6.0 | 233 | 5.2 | 204 |
| 22 | 44 | ${ }^{1793}$ | 22 | 896 | 14.7 | 598 | II.O | 448 | 8.8 | 359 | 7.3 | 299 | 6.3 | 256 | $5 \cdot 5$ | 224 |
| 23 | 46 | 1959 | 23 | 980 | 15.3 | 653 | II. 5 | 490 | 9.2 | 392 | 7.7 | 326 | 6.6 | 280 | 5.8 | 245 |
| 24 | 48 | 2134 | 24 | 1067 | 16.0 | 711 | 12.0 | 534 | 9.6 | 427 | 8.0 | 356 | 6.9 | 305 | 6.0 | 267 |
| 25 | 50 | 2315 | 25 | 1158 | 16.7 | 772 | 12.5 | 579 | 10.0 | 463 | 8.3 | 386 | 7.1 | 331 | 6.2 | 264 |
| 26 | 52 | 2504 | 26 | 1252 | 17.3 | 835 | 13.0 | 626 | 10.4 | 501 | 8.7 | 417 | 7.4 | 358 | 6.5 | 313 |
| 27 | 54 | 2700 | 27 | 1350 | 18.0 | 900 | 13.5 | 675 | 10.8 | 540 | 9.0 | 450 | 7.7 | 386 | 6.8 | $33^{8}$ |
| 28 | 56 | 2904 | 28 | 1452 | 18.7 | 968 | 14.0 | 726 | II.2 | 581 | 9.3 | 484 | 8.0 | 415 | 7.0 | 363 |
| 29 | 58 | 3115 | 29 | 1558 | 19.3 | 1038 | 14.5 | 779 | I1. 6 | 623 | 9.7 | 519 | 83 | 445 | 7.2 | 389 |
| 30 | 60 | 3333 | 30 | 1667 | 20.0 | IIII | 15.0 | 833 | 12.0 | 667 | 10.0 | 556 | 8.6 | 476 | 7.5 | 417 |

Table XI.* is such a table, computed for total centre heights from 1 to 50 feet, and for total widths from 1 to 100 feet. In railroad work neither of these quantities can be as small as one foot, but the table is designed for use in all cases where. the parallel end areas may be subdivided into an equal number of triangles or quadrilaterals.

Example 1. Three-level Ground having two Warped Surfaces.-Find the volume of two prismoids of which the following are the field-notes, the width of bed being 20 feet, and the slopes $\mathrm{I} \frac{1}{2}$ to I .

| Station 11. | $\frac{28.9 \dagger}{+12.6}$ | $\frac{0}{+18.6}$ | $\frac{43.0}{+22.0}$ |
| :--- | :---: | :---: | :---: |
| Station 12. | $\frac{27.1}{+11.4}$ | $\frac{0}{+14.8}$ | $\frac{40.3}{+20.2}$ |
| Station 12 +56. | $\frac{24.3}{+9.5}$ | $\frac{0}{\mp 10.3}$ | $\frac{34.9}{\mp 16.6}$ |

From the table, p. 410, giving values of $C_{0}$ and $K$, we find for $w=20$, and $r=1 \frac{1}{2}, C_{0}=6.7$, and $K=247$.

The computation may be tabulated as follows:

| Sta. | Width, <br> $D=d+d^{\prime}$ | Height, <br> $C=c+c_{0}$ | Partial Volume. | Volume of <br> Prismoid. |
| :---: | :---: | :---: | :---: | :---: |
| II | 71.9 | 25.3 | 562 |  |
| M | 69.6 | 23.4 | $503 \times 4=2012$ |  |
| 12 | 67.4 | 21.5 | $\frac{447}{3021}-247$ | 2774 |
| M | 63.3 | 19.2 | $374 \times 4=1496$ |  |
| 12 +56 | 59.2 | 17.0 | $\left.\frac{311}{(2254}-247\right)$ | 1124 |

* Modeled somewhat after Crandall's Tables, but adapted to give volumes by the Prismoidal Formula at once instead of by the method of mean end areas first and correcting by the aid of another table to give prismoidal volumes, as Prof. Crandall has done.
$\dagger$ The numerators are the distances out, and the denominators are the heights above grade,+ denoting cut and - fill.

Entering the table (No. XI.) for a width of 71 and a height of 25 , we find 548, to which add 7 for the 3 tenths of height, and 7 more for the 9 tenths in width, both mentally, thus giving 562 cu . yds. for this partial volume. Similarly for the width 67.4 , and height 21.5 , obtaining 447 cu . yds. The corresponding result for the middle area is 503 , which is to be multiplied by 4 , thus giving $2012 \mathrm{cu} . \mathrm{yds}$. The sum of these is 302 I cu. yds ., from which is to be subtracted the constant volume $K$, which in this case is 247 cu . yds., leaving $2774 \mathrm{cu} . \mathrm{yds}$. as the volume of the prismoid.

The next prismoid is but 56 feet long, but it is taken out just the same as though it were full, and then 56 hundredths of the resuiting volume taken. The data for the 12 th station is used in getting this result without writing it again on the page.

Example 2. Five-level Ground having four Warped Surfaces.-Find the volume of a prismoid of which the following are the field-notes, the width of bed being 20 feet, and the slopes $\mathrm{I}_{\frac{1}{2}}$ to I :

$$
\begin{aligned}
& \text { 11. } \frac{28.9}{+12.6} \quad \frac{15.0}{+12.0} \\
& \text { 12. } \frac{0}{+18.6} \\
& \frac{27.1}{+11.4}
\end{aligned} \frac{20.0}{+21.0} \quad \frac{12.5}{+12.0} \quad \frac{0}{+14.8} \quad \frac{18.5}{+19.6} \quad \frac{40.3}{+20.2}
$$

This is the same problem as the preceding, with intermediate heights added.

To compute this from the table, it is separated into three prismoids, as shown in Fig. II3.


Fig. if3.

Let $A B D G C F E$ be the cross-section. This may be separated into the triangle $A B C$, and the two quadrilaterals $B C G D$ and $A C F E$. The area of the triangle is $\frac{1}{2} c w$. That of the right quadrilateral is, from Art. 179, p. 202,

$$
\begin{aligned}
& \left.\frac{1}{2}\left[c\left(d_{k}-\frac{w}{2}\right)+k\left(d_{h}-0\right)+h\left(\frac{w}{2}-d_{k}\right)\right)\right]=\frac{1}{2}\left[(c-h)\left(d_{k}-\frac{w \prime}{2}\right)+k d_{n}\right] . \\
& \text { Similarly the area of the left quadrilateral is } \quad \frac{1}{2}\left[\left(c-h^{\prime}\right)\left(d^{\prime \prime}{ }_{k}-\frac{w}{2}\right)+k^{\prime} d^{\prime}{ }_{h}\right] . \cdot=
\end{aligned}
$$

The total area of the section then is

$$
\begin{equation*}
A=\frac{1}{2}\left[\left(c-h^{\prime}\right)\left(d^{\prime}{ }_{k}-\frac{w}{2}\right)+k^{\prime} d^{\prime}{ }_{n}+c w+k d_{n}+(c-h)\left(d_{k}-\frac{w}{2}\right)\right] . . \tag{I}
\end{equation*}
$$

If the interior side elevations be taken over the edges of the base, then $d_{k}-\frac{w}{2}$ and $d_{k}-\frac{w}{2}$ both become zero, and the first and last terms disappear. Or if the centre and extreme side heights are the same, these terms go out. Experience shows that these terms can usually be neglected without material error. If they are retained, each partial volume will be composed of five terms, while if they are neglected there will be but three. The signs of these terms also must be carefully attended to. When the interior side readings are taken over the edges of the base, therefore, this equation becomes

$$
\begin{equation*}
A=\frac{1}{2}\left(k^{\prime} d^{\prime}{ }_{h}+c w+k d_{h}\right) \tag{2}
\end{equation*}
$$

The tables are well adapted to compute the prismoidal volume for five-level sections by either of these formulæ. Thus, if the adjacent section also has five points determined in its surface, its area may be represented by an equation similar to one of these, and from these end-area data mean values may be found for the corresponding middle-area points, and the volumes taken out as before. In this case the prism included between the road-bed and side-slopes, whose volume is $K$, is not included, and hence its volume is not to be deducted from the result. The computation by table XI. of equation ( 1 ) would be as follows:

| Sta. | $h^{\prime}$. | $d^{\prime \prime}{ }^{\text {b }}$ | $k^{\prime}$. | $d^{\prime \prime}{ }^{*}$ | $c$. | $d_{k^{*}}$ | k. | $d_{h}$. | $h$. | Partial Volumes. | Total <br> Volume. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| II | 12.6 |  | 12.0 | 15.0 | 18.6 | 20.0 | 1.0 | 43.0 | 22.0 | +9+108+114+279-10 $=500$ |  |
| M |  |  | 12.0 | 13.8 | 16.7 | x9.2 |  | 4 I .6 | $21 . x$ | $4(+6+104+102+260-12)=1840$ |  |
| 12 | 11.4 | 27.1 | 12.0 | 12.5 | 14.8 | 18.5 | 19.6 | 40.3 | 20,2 | $+3+100+90+242-13=422$ | 2762 |

The use of the table is the same as before. First take out from the table the volume corresponding to $\left(c-h^{\prime}\right)\left(d^{\prime}{ }_{k}-\frac{w}{2}\right)$, which when evaluated for section II is $(18.6-12.6)(15.0-10)=6.0 \times 5.0$. This is positive, and the volume corresponding to a depth of 6.0 feet and a width of 5.0 feet is 9 cubic yards. Proceed to evaluate the remaining terms of eq. ( 1 ) in a similar manner, the last term coming out negative. The dimensions of the mid section are the means of the corresponding end dimensions, as before. If one end-area is a three-level section and the next a five-level section, the included prismoid is computed as a five-level prismoid, the vanishing points in the three-level section corresponding to the interior side elevations on the five-level section being indicated in the field. Partial stations, or prismoids, are first computed as though they were roo feet long (for which the table is constructed), and then multiplied by their length and divided by 100 as before.

If equation (2) may be used, the work is shortened very much. The columns in $h^{\prime}, d^{\prime}{ }_{k}, d_{k}$, and $h$, may be omitted, and there will also be but three terms in each partial product. Thus, if sections II and I2 had been taken with the interior elevations, each ro feet from the centre line, we might have had something as follows :

$$
\begin{aligned}
& \text { II. } \frac{28.9}{+12.6} \\
& \begin{array}{lllll}
+15.4 & \frac{10.0}{+18.6} & \frac{10.0}{+19.8} & \frac{43.0}{+22.0} \\
\text { I2. } \frac{27.1}{+I I .4} & \frac{10.0}{+12.5} & \frac{0}{+14.8} & \frac{10.0}{+17.4} & \frac{40.3}{+20.2}
\end{array}
\end{aligned}
$$

The computation then, by eq. (2), would have been :

| Sta. | $d^{\prime \prime}{ }_{h}$ | $k^{\prime}$. | $c$. | $k$. | $d_{h}$. | Partial Volumes. | Total <br> Volume. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| II | 28.9 | 15.4 | 18.6 | 19.8 | 43.0 | $137+114+263=514$ |  |
| M | $28 . \mathrm{C}$ | 14.0 | 16.7 | 18.6 | 41.6 | $4(121+102+239)=1848$ |  |
| I2 | 27.1 | 12.5 | 14.8 | 17.4 | 40.3 | $104+90+215=409$ | 2771 |

By this method the computation of a five-level section is little more trouble
than that of a three-level section, and yet the intermediate points taken at a distance of $\frac{w}{2}$ from the centre, are apt to increase the accuracy considerably on ordinary rolling ground.
321. Three-level Sections, the Surface divided into four Planes by Diagonals.-If the surface included between two three-level sections be assumed to be made up of four planes formed by joining the centre height at one end with a side, height at the other end section on each side the centre line (Fig. 114), these lines being called diagonals, an exact computation of the volume is readily made without computing the mid-area. Two diagonals are possible on each side the centre line but the one is drawn which is observed to most nearly fit the surface. They are noted in the field when the cross-sections are taken.

The total volume of such a prismoid in cubic * yards is

$$
\begin{align*}
V=\frac{l}{6 \times 27}\left[\left(d_{1}+\right.\right. & \left.d_{1}^{\prime}\right) c_{1}+\left(d_{2}+d_{2}^{\prime}\right) c_{2}+D C+D^{\prime} C^{\prime} \\
& \left.+\frac{w}{2}\left(h_{1}+h_{2}+H+h_{1}^{\prime}+h_{2}^{\prime}+H^{\prime}\right)\right],{ }^{*} \tag{I}
\end{align*}
$$

where $c_{1}, h_{1}$, and $h_{1}^{\prime}$ are the centre and side heights at one section and $d_{1}$ and $d_{1}^{\prime}$ the distances out, $c_{2}, h_{2}^{\prime}, h_{2}, d_{2}$, and $d_{2}^{\prime}$ be-

[^3]ing the corresponding values for the other end section. $C$ and $C^{\prime}$ are the centre heights, $H$ and $H^{\prime}$ the side heights, and $D$ and $D^{\prime}$ the distances out on the right and left diagonals. Although this formula seems long, the computations by it are very simple. Thus let the volume be found from the following field-notes for a base of 20 feet and side slopes $\mathrm{I}_{\frac{1}{2}}$ to I .


The upper figures indicate the distances out and those below the lines the heights, the plus sign being used for cuts. The computation in tabular form is as follows:

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline Sta. \& d. \& h. \& c. \& $h^{\prime}$. \& $d^{\prime \prime}$. \& $d+d^{\prime \prime}$. \& $\left(d+d^{\prime}\right) c_{\text {c }}$ \& $D C$. \& $D^{*} C^{*}$. <br>
\hline \multirow[t]{6}{*}{} \& 22
34 \& $$
\begin{array}{r}
8 \\
16
\end{array}
$$ \& 8 \& \& $47 \cdot 5$
16 \& 69.5
50.0 \& 556
200 \& $\cdots$ \& 128.

128 <br>
\hline \& \& \& +h \& 24 \& \& \& 88 \& \& <br>
\hline \& \& \& + ${ }^{\text {r }}$ \& 12 \& \& \& 128 \& \& <br>
\hline \& \& \& $\Sigma h '$ \& 65 \& . 0 \& \& $=650$ \& \& <br>
\hline \& \& \& \& \& \& \& 6) $\overline{162200}$ \& \& <br>
\hline \& \& \& \& \& \& \& $2 7 \longdiv { 2 7 0 3 3 }$ \& \& <br>
\hline
\end{tabular}

The great advantage of the method consists in the data all being at hand in the field-notes.

Hudson's Tables * give volumes for this kind of prismoid.

[^4]They furnish a very ready method of computing volumes whet: this system is used.
322. Comparison of Methods by Diagonals and by Warped Surfaces.-Although the surveyor has a choice oi two sets of diagonals when this method is used, the real surface would usually correspond much nearer the mean of the two pairs of plane surfaces than to either one of them. That is, the natural surface is curved and not angular, and therefore it is probable that two warped surfaces joining two three-level sections would generally fit the ground better than four planes, notwithstanding the choice that is allowed in the fitting of the planes. More especially must this be granted when the truth of the following proposition is established.

Proposition: The volume included between two three-level sections Raving their corresponding surface lines joined by warped surfaces, is exactly a mean between the two volumes formed between the same end sections by the two sets of planes resulting from the two sets of diagonals which may be drawn.

If the two sets of diagonals be drawn on each side the centre line and a crosi-section be taken parallel to the end areas, the traces of the four surface planes on each side the centre line on the cutting plane will form a parallelogram, the diagonal of which is the trace of the warped surface on this cutting plane. Since this cutting plane is any plane parallel to the end areas, and since the warped surface line bisects the figure formed by the two sets of planes formed by the diagonals, it follows that the warped surface bisects the volume formed by the two sets of planes. The proposition will therefore be established if it be shown that the trace of the warped surface is the diagonal of the parallelogram formed by the traces of the four planes formed by the two sets of diagonals. Fig. 115 shows an extreme case where the centre height is higher than the side height at one end and lower at the other. Only the left half of the prismoid is shown in the figure. The
cutting plane cuts the centre and side lines and the two diagonals in $\epsilon f g h$ on the plane, and in $e^{\prime} f^{\prime} g^{\prime} h^{\prime}$ on the vertical projection. For the diagonal $c_{1} d_{2}$ the surface lines cut out are $e^{\prime} f^{\prime}$ and $f^{\prime} h^{\prime}$. For the diagonal $c_{2} d_{1}$ they are $e^{\prime} g^{\prime}$ and $g^{\prime} k^{\prime}$. For the warped surface the line cut out is $e^{\prime} h^{\prime}$, this being an


Fig. 115.
element of that surface. It remains to show that $e^{\prime} f^{\prime} h^{\prime} g^{\prime}$ is a parallelogram.

Since the cutting plane is parallel to the end planes all the lines cut are divided proportionally. That is, if the cutting plane is one $n^{\text {th }}$ of $l$ from $c_{2}$, then it cuts off one $n^{\text {th }}$ of all the lines cut, measured from that end plane. But if the lines are divided proportionally, the projections of those lines are divided proportionally, and hence the points $e^{\prime}, f^{\prime}, h^{\prime}, g^{\prime}$ divide
the sides of the quadrilateral $d_{2}^{\prime}, c_{1}^{\prime}, c_{2}^{\prime}, d_{1}^{\prime}$ proportionally. But it is a proposition in geometry that if the four sides of a quadrilateral, or two opposite sides and the diagonals, be divided proportionally and the corresponding points of subdivision joined, the resulting figure is a parallelogram. Therefore $e^{\prime} f^{\prime} h^{\prime}$ $g^{\prime}$ is a parallelogram, and $e^{\prime} h^{\prime}$ is one of its diagonals and hence bisects it. Whence the surface generated by this line moving along $c_{1} c_{2}$ and $d_{1} d_{2}$ parallel to the end areas bisects the volume formed by the four planes resulting from the use of both diagonals on one side the centre line. Q. E. D.

It is probable, therefore, that the warped surface would usually fit the ground better than either of the sets of planes, formed by the diagonals. Furthermore, the errors caused by the use of the warped surface (Table XI.) are compensating errors, thus preventing any marked accumulation of errors in a series of prismoids.* There are extreme cases, however, such as that given in the example, Fig. II4, which are best computed by the method by diagonals.
323. Preliminary Estimate from the Profile. -If the cross-sections be assumed level transversely then for given width of bed and side slopes, a table of end areas may be prepared in terms of the centre heights. From such a table the

[^5]end areas may be rapidly taken out and plotted as ordinates from the grade line. The ends of these ordinates may then be joined by a free-hand curve, and the area of this curve found by the planimeter. The ordinates may be plotted to such a scale that each unit of the area, as one square inch, shall represent a convenient number of cubic yards, as 1000. The record of the planimeter then in square inches and thousandths gives at once the cubic yards on the entire length of líne worked over by simply omitting the decimal point. Evidently the scale to which the ordinates are to be drawn to give such a result is not only a function of the width of bed and side slopes, but also of the longitudinal scale to which the profile line is plotted. The area of a level section is
\[

$$
\begin{equation*}
A=w c+r c^{2}, . \tag{1}
\end{equation*}
$$

\]

where $w, c$, and $r$ are the width of base, centre height, and slope-ratio respectively.

Now if $h=$ the horizontal scale of the profile, that is the number of feet to the inch, and if one square inch of area is to represent 1000 cu . yards, the length of the ordinate must be

$$
\begin{equation*}
y=\frac{h A}{1000 \times} \frac{h 7}{27}=\frac{h\left(w c+r c^{2}\right)}{27,000} . \tag{2}
\end{equation*}
$$

If values be given to $h, w$, and $r$, which are constants for any given case, then the value of $y$ becomes a function of $c$ only, and a table can be easily prepared for the case in hand. Since $y$ is a function of the second power of $c$, the second difference will be a constant, and the table can be prepared by means of first and second differences. Thus if $c$ takes a small increment, as I foot, then the first difference is

$$
\begin{equation*}
\Delta^{\prime} y=\frac{h}{27,000}(w+2 r c+r) . \tag{3}
\end{equation*}
$$

But this first difference is also a function of $c$, and hence when $c$ takes an increment this first difference changes by an amount equal to

$$
\begin{equation*}
\Delta^{\prime \prime} y=\frac{h}{27000} \cdot 2 r \tag{4}
\end{equation*}
$$

which is constant. An initial first difference being given for a certain value of $c$, a column of first differences can be obtained by simply adding the $\Delta^{\prime \prime} y$ continuously to the preceding sum. With this column of first differences the corresponding column of values of $y$ may be found by adding the first differences continuously to the initial value of $y$ for that column.*

TABULAR VALUES OF $y$ IN EQUATION (2) FOR $w=20, r=\mathrm{I} \frac{1}{2}$, AND $h=400$.

| $c$ | -.'o | 0.' I | 0.12 | 0.13 | -. ${ }^{\prime} 4$ | -. ${ }^{\prime}$ | 0.6 | 0.'7 | -. ${ }^{\prime} 8$ | -.'9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | $\begin{aligned} & \text { in. } \\ & 0.00 \end{aligned}$ | $\begin{aligned} & \text { in. } \\ & 0.03 \end{aligned}$ | $\begin{aligned} & \text { in. } \\ & 0.06 \end{aligned}$ | $\begin{aligned} & \text { in. } \\ & 0.09 \end{aligned}$ | $\begin{gathered} \operatorname{in.}_{1} \\ 0.12 \end{gathered}$ | $\begin{aligned} & \text { in. } \\ & 0.15 \end{aligned}$ | $\begin{aligned} & \text { in. } \\ & \text { o. } 19 \end{aligned}$ | $\begin{aligned} & \text { in. } \\ & 0.22 \end{aligned}$ | $\begin{aligned} & \text { in. } \\ & 0.25 \end{aligned}$ | $\begin{aligned} & \text { in. } \\ & 0.28 \end{aligned}$ |
| 1 | . 32 | -35 | -39 | . 42 | . 46 | . 49 | . 53 | . 57 | .61 | . 64 |
| 2 | . 68 | . 72 | . 76 | . 80 | . 84 | . 88 | . 92 | . 96 | 1.00 | 1.05 |
| 3 | 1.09 | 1.13 | 1.17 | 1.22 | 1.26 | 1.3r | 1. 35 | 1.40 | 1.45 | 1.49 |
| 4 | x. 54 | 1.59 | 1. 63 | 1. 69 | 1. 73 | 1.78 | 1.83 | 1.88 | 1.93 | 1.99 |
| 5 | 2.04 | 2.09 | 2.14 | 2.19 | 2.24 | 2.30 | 2.36 | 2.41 | 2.47 | 3.52 |
| 6 | 2.58 | 2.63 | 2.69 | 2.75 | 2.80 | 2.87 | 2.92 | 2.98 | 3.04 | 3.10 |
| 7 | 3.16 | 3.22 | 3.28 | 3.35 | 3.41 | 3.47 | 3.54 | 3.60 | 3.66 | 3.73 |
| 8 | 3.79 | 3.86 | 3.92 | 3.99 | 4.05 | 4.13 | 4.19 | 4.26 | 4.33 | 4.40 |
| 9 | 4.47 | 4.54 | 4.60 | 4.68 | 4.75 | 4.82 | 4.89 | 4.97 | 5.04 | 5.11 |
| 10 | 5.18 | 5.26 | $5 \cdot 33$ | $5 \cdot 40$ | 5.48 | 5.56 | 5.64 | 5.72 | 5.79 | 5.87 |
| 11 | 5.95 | 6.03 | 6.10 | 6.18 | 6.26 | 6.35 | 6.43 | 6.51 | 6.59 | 6.67 |
| 12 | 6.76 | 6.84 | 6.92 | 7.00 | 7.09 | 7.18 | 7.26 | 7.35 | 7.43 | 7.52 |
| 13 | 7.61 | 7.70 | 7.78 | 7.86 | 796 | 8.05 | 8.14 | 8.23 | 8.32 | 8.41 |
| 14 | 8.50 | 8.60 | 8.68 | 8.77 | 8.87 | 8.97 | 9.06 | 9.16 | 9.25 | 9.35 |
| 15 | 9.44 | 9.54 | 9.63 | 9.73 | 9.83 | 9.94 | 10.03 | 10.13 | 10.23 | 10.33 |
| 16 | 10.43 | 10.53 | 10.62 | 10.73 | 10.83 | 10.94 | 11.04 | 11.15 | 11.25 | 11. 35 |
| 17 | 1 x .46 | 11.56 | 11.66 | 11.77 | 11.88 | 12.00 | 12.10 | 12.21 | 12.31 | 12.42 |
| 18 | 12.53 | 12.64 | 12.75 | 12.86 | 12.97 | 13.09 | 13.20 | 13.32 | 13.42 | 13.54 |
| 19 | 13.65 | 13.77 | 13.87 | 13.99 | 14.10 | 14.23 | 14.34 | 14.47 | 14.58 | 14.70 |
| 20 | 54, 85 | 14.93 | 15.04 | 15.16 | 15,29 | 15.42 | 55.53 | 15.66 | 15.78 | 15.93 |

[^6]The preceding table was constructed in this manner, for $z v=20$ feet, $r=\mathrm{I} \frac{1}{2}$; and $h=400$ feet to the inch.
324. Borrow-pits are excavations from which earth has been "borrowed" to make an embankment. It is generally preferable to measure the earth in cut rather than in fill, hence when the earth is taken from borrow-pits and its volume is to be computed in cut, the pits must be carefully staked out and elevations taken both before and after excavating. The methods given in art. 3II are well suited to this purpose, or they may be computed as prismoids by the aid of Table XI., if preferred. To use the table it is only necessary to enter it with such heights and widths as give twice the elementary areas (triangles or quadrilaterals) into which the end sections are divided, and then multiply the final result by the length and divide by 100 . The table is entered for both end-area dimensions and also the mid-area dimensions, four times this latter result being taken the same as before.
325. Shrinkage of Earthwork.-Excavated earth first increases in volume, when removed from a cut and dumped on a fill, but it gradually settles, or shrinks, until it finally comes to occupy a less volume than it formerly did in the cut. Both the amounts, initial increase, and final shrinkage depend on the nature of the soil, its condition when removed, and the manner of depositing it in place. There can therefore be no general rules given which will always apply. For ordinary clay and sandy loam, dumped loosely, the first increase is about one twelfth, and then the settlement about one sixth of this increased volume, leaving a final volume of about nine tenths of the original volume in cut.*

Thus for 100 cubic yards of settled embankment III cubic yards in cut would be required. But a contractor should have

[^7]his stakes or poles set one fifth higher than the corresponding fill, so that when filled to the tops of these, a settlement of one sixth will bring the surface to the required grade.

These changes of volume are less for sand and more for stiff, wet clay.

For rock the permanent increase in volume is from 60 to 80 per cent, the greater increase corresponding to a smaller average size of fragment.
326. Excavations under Water.-It is often necessary to determine the volume of earth, sand, mud, or rock removed from the beds of rivers, harbors, canals, etc. If this be done by soundings alone, it is likely to work injustice to the contractor, as he wouid receive no pay for depths excavated below the required limit ; and besides, foreign material is apt to flow in and partially replace what is removed, so that the material actually excavated is not adequately shown by soundings within the required limits. It is common, therefore, to pay for the material actually removed, an inspector being usually furnished by the employer to see that no useless work is done beyond the proper bounds. The material is then measured in the dumping scows or barges. The unit of measure is the cubic yard, the same as in earthwork. There are two general methods of gauging scows, or boats. One is to actually measure the inside dimensions of each load, which is often done in the case of rock, and the other is to measure the displacement of the boat, which is the more common method with dredged material. When the barge is gauged by measuring its displacement, the water in the hold must always be pumped down to a given level, or else it must be gauged both before and after loading and the depth of water in the hold observed at each gauging. A displacement diagram (or table) is prepared for each barge, from its actual external dimensions, in terms of its mean draught. There should always be four gaugings taken to determine the draught, at four symmetrically located points
on the sides, these being one fourth the length of the barge from the ends. Fixed gauge-scales, reading to feet and tenths may be painted on the side of the barge, or if it is flat-bottomed, a gauging-rod, with a hook on its lower end at the cero of the scale, may be used and readings taken at these four points. Any distortion of the barge under its load, or any unsymmetrical loading, will then be allowed for, the mean of the four gauge-readings being the true mean draught of the boat.

To prepare a displacement diagram, the areas of the surfaces of displacement must be found for a series of depths uniformly spaced. This series may begin with the depth for no load, the hold being dry. They should then be found for each five tenths of a foot up to the maximum draught. If the boat has plane vertical sides and sloped ends these areas are rectangles, and are readily computed. If the boat is modelled to curved lines, the water-lines can be obtained from the original drawings of the boat, or else they must be obtained by actual measurement. In either case they can be plotted on paper, and their areas determined by a planimeter. These areas are analogous to the cross-sections in the case of railroad earthwork, and the prismoidal formula may be applied for computing the displacement. Thus,

Let $A_{0}, A_{1}, A_{2}, A_{3}$, etc., be the areas of the displaced water surfaces, taken at uniform vertical distances $h$ apart. Then for an even number of intervals we have in cubic yards

$$
\begin{equation*}
V=\frac{h}{3 \times 27}\left(A_{0}+4 A_{1}+2 A_{2}+4 A_{3}+\ldots A_{n}\right) \tag{I}
\end{equation*}
$$

If the total range in draught be divided into six equal portions, each equal to $h$, then Weddel's Rule* would give a

[^8]nearer approximation. With the same notation as the above we would then have, in cubic yards,
\[

$$
\begin{equation*}
V=\frac{3 / 2}{10}\left[A_{0}+A_{2}+A_{4}+A_{6}+5\left(A_{1}+A_{3}+A_{5}\right)+A_{3}\right] \ldots \tag{2}
\end{equation*}
$$

\]

These rules are also applicable to the gauging of reservoirs, mill-ponds, or of any irregular volume or cavity.

After the displaced volume of water is found, the corresponding volume of earth or rock is found by applying a proper constant coefficient. This coefficient is always less than unity, and is the reciprocal of the specific gravity of the material. This must be found by experiment. In the case of soft mud it is nearly unity, while with sand and rock it is much more. When rock is purchased by the cubic yard, solid rock is not implied, but the given quality of cut or roughly-quarried rock, piled as closely as possible. When rock is excavated, solid rock is meant. A measured volume of any material put into a gauged scow will give the proper coefficient for that material. Thus if the measured volume $V^{\prime}$ give a displacement of $V$, then $\frac{V^{\prime}}{V}=C$ is the coefficient to apply to the displacement to give the volume of that material.

TABLES.

## TABLE I.

## Trigonometric Formule.

## Trigonometric Functions.

Let $A$ (Fig. 10\%) $=$ angle $B A C=\operatorname{arc} B F$, and let the radius $A F=A B=$ $A H=1$.
We then have

$$
\begin{array}{ll}
\sin A & =B C \\
\cos A & =A C \\
\tan A & =D F \\
\cot A & =H G^{\prime} \\
\sec A & =A D \\
\operatorname{cosec} A & =A G \\
\operatorname{versin} A & =C F^{\prime}=B E \\
\operatorname{covers} A & =B K=H L \\
\text { exsec } A & =B D \\
\operatorname{coexsec} A & =B G \\
\operatorname{chord} A & =B F \\
\operatorname{chord} 2 A & =B I=2 B C
\end{array}
$$



Fig. 107.

In the right-angled triangle $A B C$ (Fig. 10\%)
Let $A B=c, A C=b$, and $B C=a$.
We then have :

1. $\sin A=\frac{a}{c}=\cos B$
2. $\cos A=\frac{b}{c}=\sin B$
3. $\tan A=\frac{a}{b}=\cot B$
4. $\cot A=\frac{b}{a}=\tan B$
5. $\sec A=\frac{c}{b}=\operatorname{cosec} B$
6. $\operatorname{cosec} A=\frac{c}{a}=\sec B$
7. vers $A=\frac{c-b}{c}=\operatorname{covers} B$
$8 \operatorname{exsec} A=\frac{c-b}{b}=\operatorname{coexsec} B$
8. covers $A=\frac{c-a}{c}=\operatorname{versin} B$
9. $\operatorname{coexsec} A=\frac{c-a}{a}=\operatorname{exsec} B$
10. $a=c \sin A=b \tan A$
11. $b=c \cos A=a \cot A$
12. $c=\frac{a}{\sin A}=\frac{b}{\cos A}$
13. $a=c \cos B=b \cot B$
14. $b=c \sin B=a \tan B$
15. $c=\frac{a}{\cos B}=\frac{b}{\sin B}$
16. $a=\sqrt{(c+b)(c-b)}$
17. $b=\sqrt{(c+\alpha)(c-\alpha)}$
18. $c=\sqrt{ } a^{2}+b^{2}$
19. $C=90^{\circ}=A+B$
20. are $=\frac{a b}{2}$

TA.BLE I.-Continued.
Trigonometric Formule.

| Solution of Oblique Trlangles. |  |  |  |
| :---: | :---: | :---: | :---: |
|  | given. | sovgit. | Formules. |
| 22 | $A, B, a$ | $C, b, c$ | $\begin{gathered} C=180^{\circ}-(A+B), \quad b=\frac{a}{\sin A} \cdot \sin B, \\ c=\frac{a}{\sin A} \sin (A+B) \end{gathered}$ |
| 23 | A, $a, b$ | $B, C, c$ | $\begin{gathered} \sin B=\frac{\sin A}{a} \cdot b, \quad C=180^{\circ}-(A+B) \\ c=\frac{a}{\sin A} \cdot \sin C \end{gathered}$ |
| 24 | C, $a, b$ | $12 \sim(A+B)$ | $1 / 2(A+B)=90^{\circ}-1 / 2 C$ |
| 25 |  | $1 / 2(A-B)$ | $\tan 1 / 2(A-B)=\frac{a-b}{a+b} \tan 1 / 2(A+B)$ |
| 26 |  | $A, B$ | $\begin{aligned} & A=1 / 2(A+B)+1 / 2(A-B), \\ & B=1 / 2(A+B)-1 / 2(A-B) \end{aligned}$ |
| 27 28 |  | area | $\begin{aligned} & c=(a+b) \frac{\cos 1 / 3(A+B)}{\cos 1 / 2(A-B)}=(a-b) \frac{\sin 1 / 2(A+B)}{\sin 1 / 2(A-B)} \\ & K=1 / 2 a b \sin C . \end{aligned}$ |
| 29 | $a, b, c$ | A | $\text { Let } s=1 / 2(a+b+c) ; \sin 1 / 2 A=\sqrt{\frac{(s-b)(s-c)}{b c}}$ |
| 30 31 |  |  | $\begin{aligned} & \cos 1 / 2 A=\sqrt{\frac{s(s-a)}{b c}} ; \tan 1 / 8 A=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \\ & \sin A=\frac{2 \sqrt{s(s-a)(s-b)(s-c)}}{b c} ; \\ & \text { vers } A=\frac{2(s-b)(s-c)}{b c} \end{aligned}$ |
| 32 33 | $A, B, C, a$ | area <br> area | $\begin{aligned} & K=\sqrt{s(s-a)(s-b)(s-c)} \\ & K=\frac{a^{2} \sin B \cdot \sin C}{2 \sin A} \end{aligned}$ |

TABLE I.-Continued.
Trigonometric Fornule.

|  | general formolas. |
| :---: | :---: |
| 34 | $\sin A=\frac{1}{\operatorname{cosec} A}=\sqrt{1-\cos ^{2} A}=\tan A \cos A$ |
| 35 | $\sin A=2 \sin 1 / 2 A \cos 1 / 2 A=$ vers $4 \cot 1 / 2 A$ |
| 36 | $\sin A=\sqrt{1 / 2 \mathrm{vers} 2 A}=\sqrt{1 / 2(1-\cos 2 A)}$ |
| 2\% | $\cos A=\frac{1}{\sec A}=\sqrt{1-\sin ^{2} A}=\cot A \sin A$ |
| ¿3 | $\cos A=1-\mathrm{vers} A=2 \cos ^{2} 1 / 2 A-1=1-2 \sin ^{2} 1 / 2 A$ |
| 29 | $\cos A=\cos ^{2} 1 / 2 A-\sin ^{2} 1 / 2 A=\sqrt{1 / 2+1 / 2 \cos 2 A}$ |
| 40 | $\tan A=\frac{1}{\cot A}=\frac{\sin A}{\cos A}=\sqrt{\sec ^{2} A-1}$ |
| 41 | $\tan A=\sqrt{\frac{1}{\cos ^{2} A}-1}=\frac{\sqrt{1-\cos ^{2} A}}{\cos A}=\frac{\sin 2 A}{1+\cos 2 A}$ |
| 42 | $\tan A=\frac{1-\cos 2 A}{\sin 2 A}=\frac{\text { vers } 2 A}{\sin 2 A}=\operatorname{exsec} A \cot 1 / 2 A$ |
| 43 | $\cot A=\frac{1}{\tan A}=\frac{\cos A}{\sin A}=\sqrt{\operatorname{cosec}^{2} A-1}$ |
| 44 | $\cot A=\frac{\sin 2 A}{1-\cos 2 A}=\frac{\sin 2 A}{\operatorname{vers} 2 A}=\frac{1+\cos 2 A}{\sin 2 A}$ |
| 45 | $\cot A=\frac{\tan 1 / 2 A}{\operatorname{exsec} A}$ |
| 46 | vers $A=1-\cos A=\sin A \tan 1 / 2 A=2 \sin ^{2} 1 / 2 A$ |
| 47 | vers $A=\operatorname{exsec} A \cos A$ |
| 48 | $\operatorname{exsec} A=\sec A-1=\tan A \tan 1 / 2 A=\frac{\operatorname{vers} A}{\cos A}$ |
| 49 | $\sin 1 / 2 A=\sqrt{\frac{1-\cos A}{2}}=\sqrt{\frac{\operatorname{vers} A}{2}}$ |
| 50 | $\sin 2 A=2 \sin A \cos A$ |
| 51 | $\cos 1 / 2 A=\sqrt{\frac{1+\cos A}{2}}$ |
| 52 | $\cos 2 A=2 \cos ^{2} A-1=\cos ^{2} A-\sin ^{2} A=1-2 \sin ^{2} A$ |

## TABLE I.-Continued.

Trigonometric Formule.

## General Formulaf.

53. $\tan 1 / 2 A=\frac{\tan A}{1+\sec A}=\operatorname{cosec} A-\cot A=\frac{1-\cos A}{\sin A}=\sqrt{\frac{1-\cos \frac{A}{A}}{1+\cos }}$
54. $\tan 2 \boldsymbol{A}=\frac{2 \tan A}{1-\tan ^{2} A}$
55. $\cot .1 / 2 A=\frac{\sin A}{\operatorname{vers} A}=\frac{1+\cos A}{\sin A}=\frac{1}{\operatorname{cosec} A-\cot A}$
56. $\cot 2 A=\frac{\cot ^{2} A-1}{2 \cot A}$
57. $\operatorname{vers} 1 / 2 A=\frac{1 / 2 \operatorname{vers} A}{1+1 / 1-1 / 2 \operatorname{vers} A}=\frac{1-\cos A}{2+\sqrt{2(1+\cos A)}}$
58. vers $2 A=2 \sin ^{2} A$
59. $\operatorname{exsec} 1 / 2 A=\frac{1-\cos A}{(1+\cos A)+\sqrt{2(1+\cos A)}}$
60. exsec $2 A=\frac{\tan ^{2} A}{1-\tan ^{2} A}$
61. $\sin (A \pm B)=\sin A \cdot \cos B \pm \sin B \cdot \cos A$
62. $\cos (A \pm B)=\cos A \cdot \cos B \mp \sin A \cdot \sin B$
63. $\sin A+\sin B=2 \sin 1 / 2(A+B) \cos 1 / 2(A-B)$
64. $\sin A-\sin B=2 \cos 1 / 2(A+B) \sin 1 / 2(A-B)$
65. $\cos A+\cos B=2 \cos 1 / 2(A+B) \cos 1 / 2(A-B)$
66. $\cos B-\cos A=2 \sin 1 / 2(A+B) \sin 1 / 2(A-B)$
67. $\sin ^{2} A-\sin ^{2} B=\cos ^{2} B-\cos ^{2} A=\sin (A+B) \sin (A-B)$
68. $\cos ^{2} A-\sin ^{2} B=\cos (A+B) \cos (A-B)$
69. $\tan A+\tan B=\frac{\sin (A \cdot+B)}{\cos A \cdot \cos B}$
70. $\tan A-\tan B=\frac{\sin (A-B)}{\cos A \cdot \cos B}$

TABLE II.
For Converting Metres, Feet, and Chains.

| Metres to Feet. |  | Fett to Metres and Chains. |  |  | Chains to Feet. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Metres. | Feet. | Feet. | Metres. | Chains. | Chains. | Feet. |
| F | 3.28087 | 1 | 0.304797 | 0.0151 | 0.01 | 0.66 |
| 2 | 6.56174 | 2 | 0.609595 | . 0303 | . 02 | 1. 32 |
| 3 | 9.84261 | 3 | 0.914392 | . 0455 | . 03 | I. 98 |
| 4 | 13.12348 | 4 | 1.219189 | . 0606 | . 04 | 2.64 |
| 5 | 16.40435 | 5 | 1. 523986 | . 0758 | . 05 | 3.30 |
| 6 | 19.68522 | 6 | 1.828784 | . 0909 | . 06 | 3.96 |
| 7 | 22.96609 | 7 | 2.133581 | . 1061 | . 07 | 4.62 |
| 8 | 26.24695 | 8 | 2.438378 | . 1212 | . 08 | 5.28 |
| 9 | 29.52782 | 9 | 2.743175 | . 1364 | .09 | 5.94 |
| 10 | 32.80869 | 10 | 3.047973 | . 15 I5 | . 10 | 6.60 |
| 20 | 65.6I739 | 20 | $6.09594^{6}$ | . 3030 | . 20 | 13.20 |
| 30 | 98.42609 | 30 | 9.143918 | . 4545 | . 30 | 19.80 |
| 40 | 131.2348 | 40 | 12.19189 | . 6061 | . 40 | 26.40 |
| 50 | 164.0435 | 50 | 15.23986 | . 7576 | . 50 | 33.00 |
| 60 | 196.8522 | 60 | 18.28784 | . 9091 | . 60 | 39.60 |
| 70 | 229.6609 | 70 | 21.33581 | 1. 0606 | . 70 | 46.20 |
| 80 | 262.4695 | 80 | 24.38378 | 1.212I | . 80 | 52.80 |
| 90 | 295.2782 | 90 | 27.43175 | 1. 3636 | . 90 | 59.40 |
| 100 | 328.0869 | 100 | 30.47973 | 1.515I | 1 | 66.00 |
| 200 | 656.1739 | 100 | 60.95946 | 3.0303 | 2 | 132 |
| 300 | 984.2609 | 300 | 91.43918 | 4.5455 | 3 | 198 |
| 400 | 1312.348 | 400 | 121.9189 | 6.0606 | 4 | 264 |
| 500 | 1640.435 | 500 | 152.3986 | 7.5756 | 5 | 330 |
| 600 | 1968.522 | 600 | $182.878+$ | 9.0909 | 6 | 396 |
| 700 | 2296.609 | 700 | 213.3581 . | 10.606 | 7 | 462 |
| 800 | 2624.695 | 800 | 243.8378 | 12.121 | 8 | 528 |
| 900 | 2952.782 | 900 | 274.3175 | 13.636 | 9 | 594 |
| 1000 | 3280.869 | 1000 | 304.7973 | 15.151 | 10 | 660 |
| 2000 | 6561.739 | 2000 | 609.5946 | 30.303 | 20 | 1320 |
| 3000 | 9842.609 | 3000 | 914.3918 | 45.455 | 30 | 1980 |
| 4000 | 13123.48 | 4000 | 1219.189 | 60.606 | 40 | 2640 |
| 5000 | 16404.35 | 5000 | 1523.986 | 75.756 | 50 |  |
| 6000 | 19685.22 | 6000 | 1828.784 | 90.909 | 60 | 3960 |
| 7000 | 22966.09 | 7000 |  | 106.06 |  |  |
| 8000 | 26246.95 | 8000 | 2438.378 | 121.2I | 80 | 5280 |
| 9000 | 29527.82 | 9000 | 2743.175 | 136.36 | 90 | 5940 |

## TABLE III.

Logarithms of Numbers. § 173.


## TABLE III.-Continued.

Logarithms of Numbers.


TABLE IIIa.
Logarithms of Sines and Tangents.

|  | $0^{\circ}$ |  |  |  | $\mathrm{I}^{\circ}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Sin. | Cos. | Tan. | Cot. | Sin. | Cos. | Tan. | Cot. |  |
| $0^{\prime}$ |  | 0.0000 |  |  | 8.2419 | 9.9999 | 8.2419 | 1.7581 | $60^{\prime}$ |
| $\underline{1}$ | 6.4637 | . 0000 | 6.4637 | 3.5363 | . 2490 | . 9999 | . 249 T | . 7509 | 59 |
| 2 | $\begin{array}{r}.7648 \\ \hline .748\end{array}$ | . 0000 | $\begin{array}{r}.7648 \\ \hline .988\end{array}$ | . 2352 | . 256 r | . 9999 | .2562 | -74.38 | 58 |
| 3 | 6.9408 | . 0000 | 6.9408 | 3.0592 | . 2630 | . 9999 | . 2631 | . 7369 | 57 |
| 4 | $7.065^{8}$ | . 0000 | 70658 | 2.9342 | . 2699 | . 9999 | . 2700 | . 7300 | 56 |
| 5 | . 1627 | .0000 | . 1627 | . 8373 | . 2766 | . 9999 | .2767 | . 7233 | 55 |
| 6 | . 2419 | . 0000 | . 2419 | . 7581 | .2832 | -9999 | .2833 | .7167 | 54 |
|  | . 3088 | .0000 | . 3088 | . 6912 | . 2898 | . 9999 | . 2899 | . 7101 | 53 |
| 8 | . 3668 | . 0000 | .3668 | .6332 | .2962 | . 9999 | . 2963 | . 7037 | 52 |
| 9 | .4180 | . 0000 | . 4180 | . 5820 | . 3025 | . 9999 | - 3026 | . 6974 | 5 I |
| 10 | . 4637 | .0000 | . 4637 | .5363 | - 3088 | . 9999 | . 3089 | . 6918 | 50 |
| II | -505 | . 0000 | -5051 | . 4949 | -3150 | -9999 | . 3150 | . 6850 | 49 |
| 12 | . 5429 | . 0000 | . 5429 | . 4571 | -3210 | - 9999 | -3211 | . 6789 | 48 |
| 13 | . 5777 | . 0000 | . 5777 | . 4223 | - 3270 | . 9999 | . 3271 | . 6729 | 47 |
| 14 | .6099 | .000J | . 6099 | -3901 | - 3329 | . 9999 | -3330 | . 6670 | 46 |
| 15 | . 6398 | .000) | . 6398 | .3602 | . 3388 | . 9999 | $\cdot 3.389$ | .66ir | 45 |
| 16 | . 6678 | . 0000 | . 6678 | - 3322 | - 3445 | -9999 | . 3446 | . 6554 | 44 |
| 17 | . 6942 | . 0000 | . 6942 | - 3058 | -3502 | . 9999 | - 3503 | . 6497 | 43 |
| 18 | . 7190 | . 0000 | . 7190 | . 2810 | - 3558 | -9999 | - 3559 | . 644 r | 42 |
| 19 | .7423 | . 0000 | . 7425 | . 2575 | - 3613 | -9999 | - 3614 | . 6386 | 4 I |
| 20 | . 7648 | . 0000 | .7648 | . 2352 | . 3668 | . 9999 | . 3669 | . 6331 | 40 |
| 21 | . 7859 | . 0000 | . 7860 | . 2140 | -3722 | - 9999 | - 3723 | . 6277 | 39 |
| 22 | . 806 r | . 0000 | . 8062 | . 1938 | - 3775 | . 9999 | . 3776 | . 6224 | 38 |
| 23 | . 8255 | . 0000 | . 8255 | . 1745 | - 3828 | . 9999 | . 3829 | . 6171 | 37 |
| 24 | . 8439 | . 0000 | . 8439 | . 1561 | - $3^{880}$ | . 9999 | -3881 | . 6119 | 36 |
| 25 | . 8617 | . 0000 | .8617 | . 1383 | -3931 | - 9999 | - 3932 | . 6068 | 35 |
| 26 | . 8787 | . 0000 | . 8787 | .1213 | . 3982 | -9999 | .3983 | . 6017 | 34 |
| 27 | . 8951 | .0000 | . 895 x | . 1049 | . 4032 | . 9999 | . 4033 | . 5967 | 3.3 |
| 28 | .9109 | . 0000 | .9109 | .0891 | . 4082 | . 9999 | . 4083 | . 5917 | 32 |
| 29 | .9261 | . 0000 | . 926 I | . 0739 | .413I | . 9999 | . 4132 | . 5868 | 31 |
| 30 | . 9408 | .0000 | . 9409 | .0591 | . 4179 | . 9999 | .4181 | . 5819 | 30 |
| 31 | . 9551 | . 0000 | .9751 |  |  | . 9998 |  | . 5771 |  |
| 32 | . 9689 | .0000 | . 9689 | . 0311 | . 4275 | . 9998 | . 4276 | . 5724 | 28 |
| 33 | . 9822 | .0000 | . 9823 | .0177 | .4322 | . 9998 | . 4323 | . 5677 | 27 |
| 34 | 7.9952 | . 0000 | 7.9952 | 2.0048 | . 4368 | . 9998 | . 4370 | . 5630 | 26 |
| 35 | 8.0078 | . 0000 | 8.0078 | 1.9922 | . 4414 | . 9998 | . 4416 | . 5584 | 25 |
| 36 | .0200 | . 0000 | . 0200 | . 9800 | . 4459 | . 9098 | . 4461 | - 5539 | 24 |
| 37 | .0319 | . 0000 | . 0319 | . 968 x | . 4504 | . 9998 | . 4506 | . 5494 | 23 |
| 38 | . 0435 | . 0000 | .0433 | . 9565 | . 4549 | . 9998 | . 4551 | . 5449 | 22 |
| 39 | . 0548 | .0000 | .0548 | . 9452 | . 4593 | . 9998 | . 4595 | . 5405 | 21 |
| 40 | . 0658 | ,000 | .0658 | . 9342 | . 4637 | . 9998 | . 4638 | . 5362 | 20 |
| 4 I | . 0765 | . 0000 | . 0765 | . 9235 | . 4680 | . 9998 | .4682 | . 5318 | 19 |
| 42 | .0870 | . 0000 | . 0870 | . 9130 | . 4723 | . 9998 | . 4725 | . 5275 | 18 |
| 43 | . 0972 | . 0000 | . 0972 | . 9028 | .4765 | . 9998 | . 4767 | . 5233 | 17 |
| 44 | . 1072 | . 0000 | . 1072 | . 8928 | . 4807 | . 9998 | .4809 | . 5191 | 16 |
| 45 | . 1169 | . 0000 | . 1170 | .8830 | . 4848 | . 9998 | .4851 | . 5149 | 15 |
| 46 | . 1265 | . 0000 | .1265 | . 8735 | . 4890 | . 9998 | . 4892 | . 5108 | 14 |
| 47 | . 1358 | . 0000 | . 1359 | . 8641 | . 4930 | . 9998 | . 4933 | . 5067 | 13 |
| 48 49 | -1450 | .0000 | . 1450 | . 8550 | . 4971 | . 9998 | . 4973 | . 5027 | 12 |
| 49 50 | 1539 .1627 | .0000 | 1540 .1627 | .8460 .8373 | .5011 .5050 | . 9998 | .5013 .5053 | .4987 .4947 | 110 |
| 50 | . 1627 | .0000 | . 1627 | . 8373 | . 5050 | . 9998 | . 5053 | . 4947 | 10 |
| 51 | .1713 | .0000 | .1713 | . 8287 | . 5090 | . 9998 | . 5092 | . 4908 | 8 |
| 52 | . 1797 | 0.0000 | . 1798 | . 8202 | . 5129 | . 9998 | -5131 | . 4869 | 8 |
| 53 | . 1880 | 9.9999 | . 1880 | . 8120 | . 5167 | . 9998 | . 5170 | . 4830 | 7 |
| 54 55 | . 1961 | . 9999 | . 1962 | . 8038 | . 5206 | . 9998 | .5208 | . 4792 | 6 |
| 55 | . 2041 | . 9999 | . 2041 | . 7959 | . 5243 | . 9998 | .5246 | . 4754 | 5 |
| 56 | . 2119 | - 9999 | . 2120 | . 7880 | . 5281 | . 9998 | .5283 | .4717 | 4 |
| 57 | . 2196 | . 9999 | . 21296 | .7804 | .5318 | . 9997 | . 5321 | . 4679 | 3 |
| 58 | . 2271 | . 9999 | . 2272 | . 7728 | . 5355 | . 9997 | . 5358 | . 4642 | 2 |
| 59 60 | 8.2346 | . 9999 | . 2346 | . 7654 | 8.5392 | . 9997 | $\bigcirc .5394$ | . 4606 | 1 |
| 60 | 8.2419 | 9.9999 | 8.2419 | 1.7581 | 8.5428 | 9.9997 | 8.543 r | 1.4569 | 0 |
|  | Cos. | Sin. | Cot. | Tan. | Cos. | Sin. | Cot. | Tan. |  |
|  |  |  |  |  |  |  | $88^{\circ}$ |  |  |

TABLE IIIA.-Continued.
Logarithms of Sines and Tangents.


## TABLE IIIA-Continued.

Logarithms of Sines and Tangents.

| dre. |  |  |  |  |  | Df. |  |  | Arc. | Sin. | Df. | 3. | Df. | Tan. | Df. | Cot. | Arc. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 c | 8.9403 | 14 | 9.9983 | 1 | 8.9420 |  | . 0580 | 85 |  | 9.4130 | 47 | 9 | 3 | 9.428 I | 50 |  |  |
|  | . 9545 | 137 | .9982 | 1 | . 9563 | 138 | . 0437 | 50 | 10 | . 4177 | 46 | .9846 | 3 | 9.4281 .4331 | 50 | 0.5719 .5669 | 75 50 |
| 20 | . 9682 | 124 | . 9981 | 1 | . 9701 | I35 | . 0299 | 40 | 20 | .4223 | 46 | .9843 | 3 | . 438 r | 49 | . 5619 | 40 |
| 30 |  | 12 | . 998 | 1 | - | 1 | 4 |  | 30 | 69 | 45 | -9839 | 3 | . 4430 | 49 | . 5570 | O. |
| 4 | 8.9945 | 125 | - 9779 | 2 | 8.9966 | 1271 | 1.0034 | 20 | 40 | . 4314 | 45 | . 9836 | 3 | - 4479 | 48 | . 5521 | $0^{\circ}$ |
| 50 | 9.0070 | 122 | - 9977 | 1 | 9.0093 | 12 | .9907 |  | 50 | . 4359 | 44 | . 9832 | 4 | . 4527 | 48 | . 5473 | 10 |
| 6 | . 0192 | 119 | . 9976 | 1 | . 0216 |  | - 9784 | 840 | 160 | . 4403 | 44 | 28 |  | . 4575 |  |  |  |
|  | . 03 | 115 | . 9975 | 2 | .0336 | 117 | . 9664 | 50 | 10 | - 4447 | 44 | . 9825 | 4 | . 4622 | 47 | . 5325 | 50 |
| 20 | . 04 | 113 | . 99 | 1 | . 0 | 114 | -9547 | 40 | 20 | . 4491 | 42 | . 9821 | 4 | . 4669 | 47 | . 533 I | 40 |
| 3 |  |  | -9 | 1 |  |  | -9433 |  | 3 | . 453 | 43 | 7 | 3 | . 4716 | 46 | . 5284 | 0 |
| 40 |  | 10 | -9971 | 2 |  | 108 | . 9322 | 20 | 40 | . 4 | 42 | . 9814 | 4 | . 4762 | 46 | . 5238 | 0 |
| 5 | . 07 | 10 | - 9 | 1 | . 0 | 105 | . 9214 | 10 | 50 | . 4618 | 41 | .9810 | 4 | . 4808 | 45 | . 5192 | 10 |
| 7 |  | 102 | . 9 | 2 | .0891 | 104 |  | 83 ○ | 170 |  | 4 I |  | 4 |  | 45 | 147 | 30 |
|  | . 0 | 99 |  | 2 | . 0995 | 1 | 5 | 50 | 10 | . 4700 | 4 I | . 9802 | 4 |  | 45 |  | 50 |
| 20 | . 10 | 97 | . 9964 | 1 |  | 98 |  | 40 | 20 | . 4 | 40 | . 9798 | 4 | . 4943 | 44 | . 5057 | - |
| 30 |  | 95 |  | 2 |  | 97 |  |  | 30 | . 4 | 40 | -9794 | 4 | . 4987 | 44 | 3 | 30 |
| 4 | . 1 | 93 |  | 2 |  | 94 |  |  | 40 | . 48821 | 40 | . 9790 | 4 | . 5031 | 44 |  | 0 |
| 50 | . 1345 | 91 | -9959 | 1 | . 1385 | 93 |  |  | 50 | . 486 r | 39 | . 9 | 4 | . 5075 | 43 | . 4925 | 10 |
| 8 |  | 89 |  | 2 |  | 91 |  | 820 | 180 | . 4900 | 39 |  | 4 | 8 | 43 | 82 |  |
|  | . 1525 | 87 | -99 | 2 | . 15 | 89 | . 843 I | 50 | 10 | 9 | 38 | -9778 | 4 | . 5161 | 42 | 39 | 50 |
| 20 |  | 85 | -9 | 2 |  | 87 |  | 40 | 20 | . 4977 | 38 | - 9774 | 4 | - 5203 | 42 | . 4797 | 40 |
| 30 |  | 84 |  | 2 |  | 86 |  |  | 30 |  | 37 | -9770 | 5 | 45 | 42 | . 4755 | 30 |
| 40 |  | 82 |  | 2 | . 1831 | 84 | 8 | 20 | $4{ }^{\circ}$ | . 5052 | 38 | -9765 | 4 | 87 | 42 | 13 | 20 |
| 50 |  | 80 |  | 2 | .1915 | 82 |  |  | 50 | . 5090 | 36 | . 9761 | 4 | . 5329 | 41 | 71 | 10 |
| 9 |  | 79 |  | 2 | . 1 | 81 |  | 81 | 190 | . 5 | 37 | 7 | 5 | - 5370 | 4 I |  | 710 |
|  | . 2 | 78 | - 3944 | 2 | . 2 | 80 |  |  |  | 3 | 35 | . 9752 | 4 | 1 | 40 |  | 50 |
| 20 |  | 76 | - 9942 | 2 | . 21 | 78 |  | - | 20 | .5199 | 36 | . 9748 | 5 | 5 | 40 | 49 | - |
|  |  | 75 |  | 2 |  | 77 |  | 30 | 30 | - 5235 | 35 | 3 | 4 | 91 | 40 | . 4509 | 30 |
| 40 | . 2 | 73 |  | 2 |  | 76 | .7687 | 20 | 40 | . 5270 | 36 | -9739 | 5 | -5531 | 40 | . 4469 | 0 |
| 50 | . 23 | 73 |  | 2 |  | 74 |  |  | 50 | . 5 | 35 | . 9734 | 4 | -5571 | 40 | . 4429 | 10 |
| 10 |  | 71 |  | 3 |  | 73 |  | 800 | 20 | 1 | 34 | -9730 | 5 |  | 39 |  | $\bigcirc$ |
|  | . 2468 | 70 |  | 2 | .2536 | 73 |  | 50 | 10 | 75 | 34 | -9725 | 4 | . 5650 | 39 | . 4350 | 50 |
| 20 | . 2538 | 68 |  | 2 |  | 71 | . 7391 | 40 |  | . 5409 | 34 | . 9721 | 5 |  | 38 | . 4311 | 40 |
| 30 |  | 68 |  | 3 |  | 70 |  |  | 30 | 43 | 34 |  | 5 |  | 39 | 73 | 30 |
| 40 | . 2674 | 66 |  | 3 |  | 69 |  | 20 | 40 | . 5477 | 33 |  | 5 | -5760 | 38 | . 4234 | 20 |
| 50 |  | 66 |  | 3 |  | 68 |  |  | 50 | . $55^{10}$ | 33 |  | 4 | . 5804 | $3{ }^{3}$ | . 4196 | 10 |
| 11 |  | 64 |  | 2 |  | 66 |  | 790 | 21 |  | 33 |  | 5 |  | 37 |  | 690 |
|  |  | 64 | -9919 | 3 | . 2953 | 67 |  | 50 | 10 | . 5576 | 33 | .9697 | 5 | . 5879 | 38 | 4121 | 50 |
| 20 |  | 63 |  | 2 |  | 65 |  | 40 |  |  | 32 |  | 5 | . 5917 | 37 | . 4083 |  |
| 30 |  | 61 |  | 3 |  | 64 |  |  | 30 |  | 32 |  | 5 | - 5954 | 37 |  | 30 |
| 40 |  | 1 |  | 2 |  | 63 | .685x | 20 | 40 | . 5673 | 31 | . 9682 | 5 | . 599 r | 37 | 09 | 20 |
| 50 | -3119 | 60 |  | 3 |  | 63 |  | 10 | 50 | . 5704 | 32 | . 9677 | 5 | . 6028 | 36 | 72 | 10 |
| 12 |  | 59 |  | 3 |  | 6r |  | 78 |  |  | 31 |  |  |  | ${ }_{3} 6$ | 3936 | 68 - |
| 10 |  | 5 |  | 2 | - 3336 | 61 | . 6664 |  | 10 |  | 31 | -966 | 6 | . 6100 | 36 | 通 | 50 |
|  |  | 57 |  | 3 |  | 61 |  |  | 20 | .5798 | 30 |  | 5 |  | 36 | 84 | 40 |
|  |  | 57 |  | 3 |  | 59 |  |  | 30 |  | 3 I |  | 5 |  | 36 | 828 | 30 |
| 40 | . 3410 | 56 | . 9 | 3 | - 3517 | 59 | .6483 |  | 40 | . 5859 | 30 | . 9651 | 5 | . 6208 | 35 | - 3792 | 20 |
| 50 |  | 53 |  | 3 |  | 58 |  |  | 50 | . 5 | 30 |  | 6 | 43 | 36 | . 3757 | 10 |
| '3 |  | 54 |  | 3 | - 3634 | 57 |  | 77 ○ | 23 |  | 29 |  |  | . 6279 | 35 |  | 670 |
|  | - 3575 | 54 | . 9884 | 3 | . 3691 | 57 | . 6309 | 50 | 10 | . 5948 | 30 | . 9635 | 6 | . 6314 | 34 | . 3686 | 50 |
|  | . 3629 | 5 | . 988 x | 3 | - 3748 | 56 | . 6252 | 40 | 20 | . 5978 | 29 | .9629 | 5 | . 6348 | 35 | $\cdot 3652$ | 40 |
| 3 | -3 | 52 |  | 3 | - | 55 |  |  | 30 |  | 29 |  | 6 | . 638 | 34 | . 3617 | 30 |
| 40 | - | 52 | . 98 | 3 | . 3859 | 55 | . 6141 | 20 | 40 | . 6036 | 29 | -9618 | 5 | . 6417 | 35 | . $35^{8} 3$ | 20 |
| 50 |  | 51 |  |  | - 3914 | 54 |  | 10 | 50 |  | 28 | .9613 | 6 | . 6452 | 34 | - 3548 | 10 |
| 140 | - $3^{8} 37$ | 50 |  | 3 |  | 53 | . 6032 | 76 | 24 O | . 6093 | 28 | . 9607 |  |  | 34 | 514 | 66 - |
|  | - 3887 | 50 | . 9 | 3 | . 40 | 53 | . 5979 | 50 | 10 | .6121 | 28 | . 9602 | 6 | . 6520 | 33 | . 3480 | 50 |
|  | - 3937 | 49 | . 98 | 3 | . 4074 | 53 | . 5926 | 40 | 20 | .6149 | 28 | . 9596 | 6 | . 6553 | 34 | . 3447 | 40 |
| 30 | - 3986 |  | . 985 | 3 |  | 52 | . 5873 |  |  | . 6177 | 28 |  | 6 | . 658 | 33 | -3413 | 30 |
| 40 | . 4035 | 48 | . 9856 | 3 | . 4178 | 51 | . 5822 |  | 40 | . 6205 | 27 | .9584 | 5 | . 6620 | 34 | . 3380 | 20 |
| 50 | . 4 | 47 | . 98 |  |  | 51 | 5770 |  | 50 | .6232 | 27 | . 9579 | 6 | . 6654 | 33 | 346 | 10 |
| 15 | 9.4130 | 47 | 9. | 3 | 9.428x | 50 | 0.5719 | 75 ○ | 50 | 0.6259 | 27 | 9.9573 | 7 | . 6687 | 33 | 0.3313 | 650 |
| Arc. | Cos. | De. | SL. | D | ot. | . | Tan. |  |  | Cos. | Df. | Sin. | Df | Cot. | Df. 1 | Tan. | Arc. |

## TABLE IIIA-Continued.

Logarithms of Sines and Tangents.

| Arc. | Sin. | Df. | Cos. | Df. | Tan. | f. | Cot. | Arc. | Ar | Sin | Df. | Cos. | Dr. | Tan. | De. | Cot. | Arc. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| OT |  |  |  |  | 9.6687 |  |  |  |  |  | 18 |  |  |  | 27 |  | -\% |
| 25 | 9.62 | 27 | 9.957 | 6 | 9.6 | 32 | -.3280 | 5 | 15 | . 7 | 18 | 9.9134 .9125 | 9 | . 8479 | 27 | - 1521 .15 | 0 |
| 20 | . 63.3 | 27 | .9561 | 6 | . 6752 | 33 | . 3248 | 40 | 20 | . 7622 | 18 | .9116 | 9 | . 8506 | 27 | . 1494 |  |
| 30 | . 6 | 26 | -9555 | 6 | . 6 | 32 | . 3215 | 30 | 30 | . 7640 | 17 | .9107 | 9 | . 8533 | 26 | . 1467 | 30 |
| 40 | .6366 | 26 | . 9549 | 6 | . 6817 | 33 | . 3183 | 20 | 40 | . 7657 | 18 | . 9098 | 9 | . 8559 | 27 | . 1441 |  |
| 50 | . 6392 | 26 | . 9543 | 6 | . 6850 | 32 | -3150 | 10 | 50 | .7675 | 17 | . 9089 | 9 | . 8586 | 27 | . 1414 |  |
| 260 | . 6 | 26 | . 9537 | 7 | . 6882 | 32 |  | 64 | 36 o | . 7692 | 18 | $\bigcirc$ | 10 |  | 26 |  |  |
| 10 | . 6 | 26 | . 95 | 6 |  | 32 | . 3086 | 50 | 10 | . 7 | 17 |  | 9 | 8666 | 27 | 61 | 50 |
| 20 | . 6470 | 25 | . 9524 | 6 | . 6946 | 31 | . 3054 | 40 | 20 | . 7727 | 17 | . 906 I | 9 | . 8666 | 26 | . 1334 | 40 |
| 3 |  | 26 | . 95 | 6 |  | 32 | -3 | 30 | 30 | - 7744 | 17 | 52 | 10 | . 8692 | 26 | . 1308 | 30 |
| 40 | . 6521 | 25 | . 95 | 7 | - 7 | 31 | . 2991 | 20 | 40 |  | 17 | -9042 | 9 | . 8718 | 27 | . 1282 |  |
| 50 | . 6546 | 24 | -9505 | 6 | -7 | 32 | . 2960 | 10 | 50 | . 7778 | 17 | $\cdot 9033$ | 10 | . 8745 | 26 | . 1255 | 10 |
| 27 | . 6 | 25 | -9 | 7 | -7 | 31 |  | 63 - | 37 O | 5 | 16 | 23 | 9 | 1 | 26 | 1229 | 530 |
| 7 |  | 25 | -9492 | 6 | .7103 | 3 I | . 2897 | 50 | 10 | -7811 | 17 | . 9014 | 10 | . 8797 | 27 | .1203 | 50 |
| 20 | . 6620 | 24 | . 9486 | 7 | . 7134 | $3{ }^{1}$ | . 2866 | 40 | 20 | . 7828 | 16 | . 9004 | 9 | . 8824 | 26 | . 1176 |  |
| 3 |  | 24 | -9 | 6 | - 7 | $3{ }^{1}$ |  | 30 | 30 |  |  |  | 10 | . 8 | 26 |  | 30 |
| 4 | . 66 | 24 | - 9 | 7 | . 7196 | 30 | . 2804 | 20 | 40 | .7861 | 16 | . 8985 | 10 | . 8876 | 26 | . 1 |  |
| 50 | . 6692 | 24 | - 9 | 7 | . 7226 | 31 | . 2774 | 10 | 50 | .7877 | 16 | . 8975 | 10 | . 8902 | 26 | . 1098 | 10 |
| 28 o |  | 24 | -9 | 6 |  | 30 | . 2743 | 62 | 38 | . 7 | 17 | 5 | 10 | . 8928 | 26 |  | 520 |
| 10 |  | 23 | . 9 | 7 | -7287 | 30 | .2713 | 50 | 10 | -7910 | 16 | . 8955 | 10 | 4 | 26 |  | 50 |
| 20 | . 6 | 24 | . 9446 | 7 | .7317 | 3 I | . 2683 | 40 | 20 | . 7926 | 15 | . 8945 | 10 | O | 26 | 1020 | - 40 |
| 3 |  | 23 | -9 | 7 | . 7348 | 30 |  | 30 | 30 | . 7941 | 16 |  | 10 | . 9006 | 26 |  | 30 |
| 40 | . 681 | 23 | . 9 | 7 | . 7378 | 30 | 2 | 20 | 40 | . 7957 | 16 | . 8925 | 10 | 32 | 2 f | . 0968 | 20 |
| 50 | . 68 | 23 | . 9425 | 7 | . 7408 | 30 | . 2592 |  | 50 | . 7973 | 16 | . 8915 | 10 | . 9058 | 26 | . 0942 | 10 |
| 29 | . 6 | 22 | -9 | 7 |  | 29 | . 2562 | 6 r | 39 |  | 15 |  | 10 | 84 | 26 |  |  |
| 10 | . 6878 | 23 | . 94 | 7 |  | 30 | . 2533 | 50 | 10 |  | 16 |  | II | 10 | 25 |  | O |
| 20 | . 69 | 22 | . 9 | 7 | . 7497 | 29 | . 2503 | 40 | 20 | . 8020 | 15 | . 8884 | 10 | .9135 | 26 |  | 40 |
| 3 |  | 23 |  | 7 |  | 30 |  |  | 30 |  | 15 |  | 10 | 61 | 26 |  | 30 |
| 40 |  | 22 | . 9 | 7 | -7556 | 29 | . 2444 | 20 | 40 | . 8050 | 16 | . 8884 | 11 | .9187 | 25 | . 0813 | 20 |
| 50 | . 6958 | 22 | .9383 | 8 | . 7585 | 29 | . 2415 | 10 | 50 |  | 15 | . 8853 | 10 | . 9212 | 26 | . 0788 | 10 |
| 30 |  | 22 |  | 7 |  | 30 |  | 60 | 40 O |  | 15 |  | 11 | 8 | 26 |  |  |
|  | . 7 | 21 | -9 | 7 |  | 29 | . 2356 | 50 | 10 |  | 15 | . 8832 | II | 264 | 25 | 36 | 50 |
| 2 | - 7 | 22 | . 9 | 8 |  | 28 | . 2327 | 40 | 20 |  | 14 | .8821 | 11 | .9289 | 26 |  | 40 |
| 30 |  | 21 | -9 | 7 |  | 29 |  | 30 | 30 |  | 15 | - | 0 | $3^{15}$ | 26 | 5 | 0 |
| 40 |  | 21 | . 9 | 8 | -7 | 29 | . 2270 | 20 | 40 | . 8140 | 15 | . 8800 | II | . 9341 | 25 |  | 20 |
| 50 | . 7097 | 21 | . 93 | 7 | -7 | 29 | . 2241 | 10 | 50 | .8r55 | 14 | . 8789 | 11 | . 9366 | 26 |  |  |
| 37 |  | 21 | .9331 | 8 |  | 28 |  |  | 410 |  | 15 |  | II | 2 | 25 | . 0608 |  |
| 10 |  | 21 | . 9323 | 8 |  | 29 | . 2184 | 50 | 10 | . 8184 | 14 | 8767 | 11 | . 9417 | 26 |  | 50 |
| 20 |  | 21 | .9315 | 7 | .7845 | 28 | . 2155 | 40 | 20 | .8198 | 15 | . 8756 | II | . 9443 | 25 | . 0557 | 40 |
| 3 |  | 20 |  | 8 |  | 29 |  | 30 |  |  | 14 | 45 | 12 | . 9468 | 26 | . 0532 | 30 |
| 4 |  | 21 | . 93 | 8 | . 7902 | 8 | . 2098 | 20 | 40 | . 8227 | 14 | . 8733 | 11 | . 9494 | 25 | . 0506 |  |
| 50 |  | 20 | . 9 | 8 | . 7930 | 28 |  |  |  |  | 14 | . 8722 | II | .9519 | 25 | . 0481 |  |
| 32 |  |  |  | 8 |  | 28 |  | 58 | 42 |  | 4 |  | 12 | 544 | 26 |  | 48 - |
|  |  | 20 |  | 8 |  | 28 |  | 50 | 10 | . 8269 | 14 | . 868 | 11 | . 9570 | 25 | . 0430 | 50 |
|  | .7282 | 20 |  | 8 |  | 28 |  | 40 |  | . 8283 | 14 | . 8688 | 12 | . 9595 | 26 | . 0405 |  |
| 3 |  | 20 | . 9 | 8 |  | 28 |  |  | 30 |  | 14 | . 8676 | II | 21 | 25 |  |  |
| 4 | -7 |  | .9252 | 8 | . 8 | 27 | . 1930 | 20 | 40 | .83II | 13 | . 8665 | 12 | . 9646 | 25 | . 0354 |  |
| 50 | . 7342 | 19 | . 9244 | 8 | . 8 | 28 | . 1903 | 10 | 50 | . 8324 | 14 | . 8653 | 12 | . 9671 | 26 | . 0329 |  |
| 33 |  | 19 |  | 8 |  | 28 |  | 570 | 430 |  | 13 | . 864 I | 2 | 97 | 5 |  |  |
|  | -7 | 20 | . 9228 | 9 | . 8153 | 27 | . 1847 | 50 | 10 | . 8351 | 14 | . 8629 | 11 | . 9722 | 25 | . 0278 | 50 |
| ${ }^{2}$ |  | 19 | .9219 | 8 |  | 28 | . 1820 | 40 | 20 | .8365 | 13 | . 8 | 12 | . 9747 | 25 | . 0253 | 40 |
| 3 |  | 9 |  | 8 |  | 27 |  | 30 | 30 | . 837 | 13 | . 8606 | 12 | . 9772 | 26 | 28 |  |
| 40 | . 7438 | 19 | .9203 | 9 | . 8 |  | .1765 | 20 | 40 | . 8391 | 14 | . 8594 | 12 | . 9798 | 25 | . 0202 |  |
| 50 | -7 | - | . 9194 | 8 |  | 27 | . 1737 | 10 | 50 | . 8405 | 13 | . 8582 | 13 | .9823 | 25 | . 0177 |  |
| 340 | . 7 | 18 |  |  |  | 27 | . 1710 | 56 o | 44 O |  | 13 |  | 12 | 848 | 26 |  | 460 |
|  | . 7494 | 19 | . 9177 | 8 | .8317 | 27 | . 1683 | 50 |  | . 843 I | 13 | . 8557 | 12 | . 9874 | 25 | . 0126 | 50 |
|  | -7513 | 18 |  | 9 | . 8344 | 27 | . 1656 | 40 | 20 |  | 13 | . 8545 | 13 | . 9899 | 25 |  | 40 |
|  | -7531 | 19 | .93 | 9 | . 8371 | 27 |  | 30 | 30 | . 8 | 12 | . 8532 | 12 |  | 25 |  |  |
|  | -7550 | 18 | . 9151 | 9 | . 8398 | 27 | 1602 | 20 | 40 | . 8469 | 13 | . 8520 | 13 | . 9949 | 26 | . 0051 | 20 |
| 50 | . 75 |  | . 9142 | 8 |  | 27 | 15 | 10 | 50 |  | 13 | . 8507 |  | 9.9975 | 25 | . 0025 | 10 |
| 350 | 9.7 | 18 | 9.9134 | 9 | 9.8452 | 27 | 0.1548 | 55 |  | . 849 |  | 9.8495 |  |  |  | 0.0000 | 45 |
| Arc. | Cos. | Df | Sin. | D | Cot. | Df | Tan. | Arc | Arc | Cos. | Df. | Sin. | De. | Cot. | Df. | Tan. | Arc. |

TABLE IV.
Logarithmic Traverse Table. § 173.

| Zero angle at South Point, and increasing to W. $\left(90^{\circ}\right)$, N. ( $180^{\circ}$ ), E. $\left(270^{\circ}\right)$. |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Arc rst and 3 d. Quadrants. | Log. sin. (Dep.) | Log. cos. (Lat.) | Arc 2d and 4th. Quadrants. | Arc ist and $3^{\text {d. }}$ <br> Quad- <br> rants. | $\begin{aligned} & \text { Log. } \\ & \sin . \\ & \text { (Dep.) } \end{aligned}$ | Log. cos. (Lat.) | Arc 2d and 4th. Quadrants. | Arc rst and 3d. Quadrants. | $\begin{aligned} & \text { Log. } \\ & \text { sin. } \\ & \text { (Dep.) } \end{aligned}$ | Log. cos. (Lat.) | Arc 2d and 4th. Quadrants. |
| $0^{\circ} 180^{\circ}$ |  | 10.0000 | $180^{\circ} 360^{\circ}$ | $1^{\circ} 181{ }^{\circ}$ | 8.2419 | 9.9999 | $179{ }^{\circ} 359^{\circ}$ | $2^{\circ} 182^{\circ}$ | 8.5428 | 9.9997 | $178{ }^{\circ} 358{ }^{\circ}$ |
| 1 2 | 6.4637 .7648 | . .0000 | 59 58 58 |  | .2490 .2561 | .9999 .9999 | 59 58 58 | 18 2 | .5464 .5500 | .9997 | 59 58 58 |
| 3 | 6.9404 | . 0000 | 57 | 3 | . 2630 | -9999 | 57 | 3 | -5535 | -9997 | 57 |
| 4 | 7.0658 | . 00000 | - 56 | 4 | . 2699 | -9999 | 56 | 4 | . 5571 | -9997 | 56 |
| - ${ }_{6}$ - | .1627 .2419 | . 00000 | - ${ }_{54}$ - | $5^{--}$ | . 2768 | . 99999 | - ${ }_{54}$ - | - 5 - | . 5605 | . 99997 | -55- |
| 7 | . 3088 | . 0000 | 53 | 7 | 2898 | . 9999 | 53 | 7 | . 5674 | .9997 | 54 53 |
| 8 | . 3668 | . 0000 | 52 | 8 | . 2962 | . 9999 | 52 |  | . 5708 | . 9999 | 52 |
| 9 | .4180 | . 0000 | 51 | 9 | -3025 | -9999 | 51 | 9 | . 5742 | -9997 | 51 |
| 10 | 7.4637 | 10.0000 | 50 | 10 | 8.3088 | 9.9999 | 50 | 10 | 8.5\%\%6 | 9.999\% | 50 |
| 11 | . 505 r | . 0000 | 49 | 11 | -3150 | . 9999 | 49 | 11 | .58n9 | -9997 | 49 |
| 12 | . 5429 | . 0000 | 48 | 12 | - 3210 | . 9999 | 48 | 12 | . 5842 | -9997 | 48 |
| 13 | . 5777 | 0000 .0000 | 47 | 13 | -3270 | . 9999 | 47 | 13 | - 5875 | .9997 | 47 |
| 14 | . 6099 | . 0000 | - ${ }^{46}$ - | - 14. | -3229 |  | - ${ }^{46}$ - | 14 -15 | -5907 | -9997 | - ${ }^{46}$ |
| - 15 - | . 6398 | . .0000 | -45- | - 15 - | - 33445 | . 99999 | -45 - | - 15 - | . 593972 | .9997 | -45 - |
| 17 | . 6942 | . 0000 | 13 | 17 | -3502 | . 9999 | 43 | 17 | . 6003 | . 9997 | 43 |
| 18 | . 7190 | . 0000 | $\therefore$ | 18 | -3558 | . 9999 | 42 | 18 | . 6035 | -9996 | 42 |
| 19 | .7425 | . 000 | 4 | 19 | -3613 | -9999 | 41 | 19 | . 6066 | -9996 | 41 |
| 20 | 7.7648 | 10.0000 | 46 | 20 | 8.3668 | 9.9999 | 40 | 20 | 8.609\% | 9.9996 | 40 |
| 21 | . 7859 | . 0000 |  | 21 | - 3722 |  |  | 21 | . 6 r 28 |  |  |
| 22 | . 80625 | . .00000 | 38 37 | 22 23 | -3775 | .9999 | $38$ | 22 | $\begin{array}{r}.6159 \\ .6189 \\ \hline\end{array}$ | . 99996 | 38 |
| 23 24 | .8255 | . .0000 | 37 36 | 23 24 24 |  | . 99999 | 37 36 | 23 24 24 | .6189 .6220 | . 99996 | 37 36 3 |
| -25- | .8617 | . 0000 | - 35 - | - 25 - | -3931 | .9999 | - 35 - | - 25. | . 6250 | . 9996 | - 35 - |



4
TABLE IV.-Continued.
Zero angle at South Point, and increasing to W. $\left(90^{\circ}\right)$, N. $\left(180^{\circ}\right)$, E. $\left(270^{\circ}\right)$.

| Arc rst and 3d. Quadrants. | $\begin{aligned} & \text { Log. } \\ & \text { sin. } \\ & \text { (Dep.) } \end{aligned}$ | $\begin{aligned} & \text { Sin. } \\ & \text { Dif. } \\ & \text { for } \\ & \text { for } \end{aligned}$ | Log. cos. <br> (Lat.) | Arc 2d and 4th. Quadrants. | Arc <br> ist and 3d. <br> Quadrants. | $\begin{aligned} & \text { Log. } \\ & \text { sin. } \\ & \text { (Dep.) } \end{aligned}$ | Sin. <br> Dif. <br> for $\mathbf{r}^{\prime}$. | Log. cos. (Lat.) | Arc 2 d and 4 th. Quadrants. | Arc ist and 3d. Quadrants. | $\begin{aligned} & \text { Log. } \\ & \text { sin. } \\ & \text { (Dep ) } \end{aligned}$ | $\begin{aligned} & \text { Sin. } \\ & \text { Dif. } \\ & \text { for } \\ & \mathbf{I}^{\prime} . \end{aligned}$ | Log. cos. (Lat.) | Arc 2d and 4th. Quadrants. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $3^{\circ} 183^{\circ}$ | 8.7188 |  |  | $177^{\circ}$ | $17^{\circ} 197^{\circ}$ | 9.4659 |  | 9.9806 | 163 | $31^{\circ} 211^{\circ}$ | 9.7118 |  | 9.9331 | ${ }^{\circ}$ |
| 10' | $.7423$ | 23 | $\text { . } 9993$ | $50^{\circ}$ | $10^{\prime}$ | -4700 | 4.1 | . 9802 | $50^{\prime}$ | $10^{\prime}$ | - 71 | 2.1 | . 93323 | $50^{\prime}$ |
| 20 | -76 | 21. | -9993 | ${ }^{40}$ |  | -474I | 4.0 | -9798 |  |  | .7160 |  | -9315 | 40 |
| 40 | .785 | 20.2 | . 99992 | 30 20 | 30 | -4781 | 4.0 | -9794 | - 30 - | - $30-$ | .7181 | 2.0 | -9308 | 30 |
|  | . 8251 | 18. | . 99 |  |  | . 486 r | 4.0 | . 97986 |  |  | . 7222 | 2.1 | .9300 |  |
| $4{ }^{\circ} 18$ | 8.8436 |  | 9.99 | $176^{\circ} 356^{\circ}$ | $18^{\circ} 198{ }^{\circ}$ | 9.4900 | 3.9 | 9.9782 | $162^{\circ} 342^{\circ}$ | $32^{\circ} 212^{\circ}$ | 9.7242 | 2.0 | 9.9284 | 328 ${ }^{\circ}$ |
| 10 | . 8613 |  |  | 50 | Io | . 4939 | 3.9 3.8 | -9778 | 168 | 10 | . 7262 .7282 |  | 9.9224 .9276 | ${ }_{50}$ |
| 20 | . 878 |  | . 9988 | 40 -30 | ${ }^{20}$ | - 4977 | 3.8 3.8 | -9 | - ${ }^{\circ} \mathrm{O}$ | - 20 | . 7282 | 2.0 | . 9268 | 40 |
| 30 | . 894 | 15.8 |  | 30 20 | 30 | . 5015 | 3.8 3.7 | -9770 | 30 | - 30 | . 730 | 2.0 2.0 | .9260 | 30 |
| 40 50 |  | 15.2 | . 9985 | 10 | 50 | . 5052 | 3.8 | .9765 |  | 40 50 | .7322 .7342 | 2.0 | . 9252 | 20 |
| $5 \cdot 185$ | 8.9403 | 14 | 9.9983 | $175{ }^{\circ} 355^{\circ}$ | $19^{\circ} 199^{\circ}$ | 9.5126 | 3.6 | 9.9757 | $161{ }^{\circ} 341{ }^{\circ}$ | $33^{\circ}{ }^{\text {213 }}{ }^{\circ}$ | 9.7361 | 1. |  | $7^{\circ}$ |
| 10 | . 9545 |  | . 9982 | 50 | 10 | 9.51563 .515 | 3.7 | -9752 | 50 | 10 | 9.7381 .7380 | 1.9 | 9.9228 | 3ス |
| 20 | . 96 |  |  | $4{ }^{\circ}$ | 20 | . 5199 | 3.6 | . 97 | 40 | =0 | . 74 |  | 9219 | 40 |
| 30 | $8.988{ }^{\text {8 }}$ | 1 |  | 30 | 30 | . 5235 | 3.6 3.5 | -97 | $-30$ | 30 | - 74 |  | . 9211 | - 30 |
| 40 | 8.9945 | 12 | -99 | ${ }^{20}$ | 40 | . 5270 | 3.6 3.6 | -97 |  | 40 | . 74 |  | . 9203 | 20 |
| $6^{\circ} \stackrel{50}{186}$ | 9.0070 | 12 | 9 | $174{ }^{\circ}$ |  | . 5306 | 3.6 3.5 | -9734 | $160^{\circ}{ }^{10} 340^{\circ}$ | 34 | . 74 |  | . 9194 |  |
| 10 | -03II | 11 | 9.9975 | 1845 | 10 | $\begin{array}{r}\text { 9. } \\ \text {. } 5334 \mathrm{I} \\ \hline\end{array}$ | $3 \cdot 4$ | 9.9730 .9725 |  | 34. | 9.74 .7 | 1.8 | 9. |  |
| 20 | . 04 | It | -9973 | 40 | 20 | - 5409 | 3.4 3.4 | .9721 | 40 | 20 | -.7494 | 1.9 | .9169 |  |
| 30 | . 053 |  | . 9972 | 30 | 30 | - 5443 |  | -9716 | 30 | 30 | -7531 |  | .9160 | - 30 |
| 40 | . 0648 | 10.7 | -9971 | ${ }^{20}$ | 40 | - 5477 |  | .9711 | 20 | 40 | . 75 | 1.8 | .9151 | 20 |
| 7. ${ }^{50} 8{ }^{\prime}$ | . 0755 | 10 |  |  | - | . 5510 |  | . 9706 | $159^{\circ} 339{ }^{\circ}$ | $35^{\circ}$ | -75 | 1.8 | .9142 |  |
| 10 |  | 10.2 |  |  |  | 9.5 | $3 \cdot 3$ | 9.9702 | $159^{\circ} 339^{\circ}$ |  |  | 8 | 9.9134 | $5^{\circ}$ |
| 20 | . 106 | 9.9 9.7 |  | 40 | 20 | . 56509 | 3 |  | 40 | O |  | 1.8 |  |  |
| 30 | . 1157 | . 7 | -9963 | 30 | - 30 | . 5641 | 3.2 3.2 | . 96 | 30 | - 30 | . 7640 | 1. | . 9107 |  |
| 40 | . 1252 |  | -996ı | 20 | 40 | . 5673 | 3.2 | . 9682 | 20 | 40 | . 7657 | 8 | .9098 |  |
| $8{ }^{-188}$ | . 1345 |  | 995 |  |  | . 5704 | 3.1 3.2 3 | . 9677 |  |  | . 7675 |  | . 9089 |  |
| $8 \cdot 188$ | 9.1436 | 8.9 | 9.9958 | $2^{\circ} 352^{\circ}$ | $22^{\circ} 20$ | 9.57 | 3.1 <br> 3.1 | 9.9 | $158{ }^{\circ} 338{ }^{\circ}$ | ${ }^{\circ} 21$ | 9.7692 | 1.8 | 9.9080 | $144^{\circ} 324{ }^{\circ}$ |
| 12 | . | 8.7 | 956 | 50 | 10 | - 5 |  | . 9667 | 50 | 10 | $\cdot 77$ |  | . 9070 | 50 |
|  |  | 8. | - | 40 | 20 |  |  |  | 40 | 20 | -7727 |  | 9061 | 40 |
| 40 | .1781 |  | . 9 | 20 |  |  | 3.1 |  | 20 |  |  |  | 9052 |  |
|  | . 1863 |  | . 9948 | 10 |  | . 5889 | 3.1 3.0 | . 9646 |  |  | . 7778 | 1.7 | 9033 |  |
| $19^{\circ} 189$ | 9.1943 | 8. | 9.994 |  | 203 | 9.5919 | 3.0 | 9.9640 | $15 \%^{\circ} 33$ | $37^{\circ} 217^{\circ}$ | 9.7795 | 1.7 | 0.9023 | $3^{\circ} 323{ }^{\circ}$ |







TABLE IV.-Continwed.

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TABLE IV.-Continued.
Logarithmic Traverse Table.

| Arc rst and 3 d Quadrants. | Log. sin. (Dep.) | Log. cos. (Lat.) | Arc 2d and 4th Quadrants. | Arc rst and $3^{\text {d }}$ Quadrants. | Log. sin. (Dep.) | Log. cos. (Lat.) | Arc 2d and 4th Quadrants. | Arc rst and 3 d Quadrants. | Log. sin. (Dep.) | Log. cos. (Lat.) | Arc 2 d and 4 th Quadrants. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $87^{\circ} 26 \%^{\circ}$ | 9.9994 | 8.7188 | $93^{\circ} 273{ }^{\circ}$ | $88^{\circ} \mathbf{2 6 8}^{\circ}$ | $\mathbf{9 . 9 9 9 7}$ | 8.5428 | 920 $2^{\circ} 2^{\circ}$ | $89^{\circ} \mathbf{2 6 9}{ }^{\circ}$ | 9.9999 | 8.2419 | $91^{\circ} 271^{\circ}$ |
| 1 2 | .9994 .9994 | .7164 .7140 | 59 58 58 | 1 2 | .9997 .9997 | .5392 .5355 | 59 58 58 | 1 <br> $2^{\prime}$ | .9999 .9999 | .2346 .2271 | 59 58 58 |
| 3 | . 9994 | . 7115 | 57 | 3 | . 9997 | . 5318 | 57 | 3 | . 9999 | . 2196 | 57 |
| 4 | . 9994 | .7090 | 56 | 4 | . 9998 | .5281 | 56 | 4 | . 9999 | . 2119 | 56 |
| 5 - | . 9994 | . 7066 | - 55 - | 5 - | . 9998 | . 5243 | - 55 - | - 5 - | . 9999 | . 2041 | - 55 - |
| 6 | -9994 | -7041 | 54 | 6 | . 9998 | . 5206 | 54 | 6 | . 9999 | . 1961 | 54 |
| 7 | . 9994 | .7016 | 53 | 7 | . 9998 | . 5167 | 53 | 7 | 9.9999 | . 1880 | 53 |
| 8 | . 9995 | . 6991 | 52 | 8 | . 9998 | . 5129 | 52 | 8 | 10.0000 | -1797 | 52 |
| 9 | .9995 | . 6965 | 51 | 9 | .ç998 | .5090 | 51 | 9 | . 0000 | .1713 | 51 |
| 10 | 9.9995 | 8.6940 | 50 | 10 | 9.9998 | 8.5050 | 50 | 10 | 10.0000 | 8.1627 | 50 |
| 11 | . 9995 | . 6914 | 49 | 11 | . 9998 | -5011 | 49 | 11 | . 0000 | . 1539 | 49 |
| 12 | . 9995 | . 6889 | 48 | 12 | . 9998 | . 4971 | 48 | 12 | . 0000 | . 1450 | 48 |
| 13 | . 9995 | . 6863 | 47 | 13 | . 9998 | . 4930 | 47 | 13 | . 0000 | . 1358 | 47 |
| 14 | . 9995 | . 6837 | 46 | 14 | . 9998 | . 4890 | 46 | 14 | . 0000 | . 1265 | 46 |
| -15 - | . 9995 | .6810 | - 45 - | - 15 - | . 9998 | .4848 | - 45 - | - 15 - | . 0000 | .1169 | - 45 - |
| 16 | . 9995 | . 6784 | 44 | 16 | . 9998 | .4807 | 44 | 16 | . 0000 | . 1072 | 44 |
| 17 | . 9995 | . 6758 | 43 | 17 | . 9998 | . 4765 | 43 | 17 | . 0000 | . 0972 | 43 |
| 18 | . 9995 | . 6731 | 42 | 18 | . 9998 | .4723 | 42 | 18 | . 0000 | . 0870 | 42 |
| 19 | . 9995 | . 6704 | 41 | 19 | . 9998 | . 4680 | 41 | 19 | . 0000 | .0765 | 41 |
| 20 | 9.9995 | 8.667\% | 40 | 20 | 9.9998 | 8.4637 | 40 | 20 | 10.0000 | 8.0658 | 40 |
| 21 | . 9995 | . 6650 | 39 | 21 | . 9998 | . 4593 | 39 | 21 | . 0000 | . 0548 | 39 |
| 22 | . 9995 | . 6622 | $3^{8}$ | 22 | . 9998 | . 4549 | 38 | 22 | . 0000 | . 0435 | 38 |
| 23 | . 9995 | . 6595 | 37 | 23 | . 9998 | . 4504 | 37 | 23 | . 0000 | .0319 | 37 |
| 24 | . 9996 | . 6567 | 36 -35 | 24 | . 9998 | . 4459 | 36 -35 | 24 -25 | . 0000 | 8.0200 | 36 -35 |
| - 25 - | . 9996 | . 6539 | - 35 - | $-25-$ | . 9998 | . $44{ }^{18}$ | - 35 - | -25- | . 0000 | 8.0078 | - 35 - |


| MMNM |  <br> 1 |  \| | $\begin{array}{ccc} 1 & \stackrel{0}{2} \\ 1 & \stackrel{0}{2} \\ 1 & \circ \\ 0 \end{array}$ |
| :---: | :---: | :---: | :---: |
|  |  |  |  © ! M Mo Mo ํ, |
| $\begin{array}{r} 8888 \% \\ 8080 \\ 0.80 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$ |  |  |  |
|  |  1 |  |  |
| $\underset{m M N M O}{\infty}$ |  \| |  |  |
|  |  $\operatorname{mon}^{\infty}{ }^{\infty} \mathrm{m}_{\infty}^{\infty}{ }^{\infty}$ NN <br>  $\infty$ |  |  |
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| 수ำค |  <br> I |  |  |

TABLE V.
Horizontal Distances and Elevationg from Stadia Readings. § 204.

| Minutes. | $0^{\circ}$ |  | $1{ }^{\circ}$ |  | $2^{\circ}$ |  | $3^{\circ}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Hor. Dist. | Diff. <br> Elev. | Hor. Dist. | Diff. <br> Elev. | Hor. <br> Dist. | Diff. <br> Elev. | Hor. <br> Dist. | Diff. <br> Elev. |
| $\bigcirc$ - | 100.00 | 0.00 | 99.97 | 1.74 | 99.88 | 3.49 | 99.73 | 5.23 |
| 2 . | " | 0.06 | " | 1.80 | 99.87 | $3 \cdot 55$ | 99.72 | 5.28 |
| 4 . | " | 0.12 | " | 1.86 | " | 3.60 | 99.71 | $5 \cdot 34$ |
| 6 . | " | 0.17 | 99.96 | 1.92 | " ${ }^{\prime}$ | 3.66 | " | $5 \cdot 40$ |
| 8 . | " | 0.23 | " | 1.98 | 99.86 | 3.72 | 99.70 | $5 \cdot 46$ |
| 10. | " | 0.29 | " | 2.04 | " | 3.78 | 99.69 | $5 \cdot 52$ |
| 12. | " | 0.35 | " | 2.09 | 99.85 | 3.84 | " | $5 \cdot 57$ |
| 14. | " | 0.41 | 99.95 | 2.15 | " | 3.90 | 99.68 | 5.63 |
| 16. | " | 0.47 | " | 2.21 | 99.84 | 3.95 | " | 5.69 |
| 18. | " | 0.52 | " | 2.27 | , | 4.01 | 99.67 | 5.75 |
| 20. | " | 0.58 | " | 2.33 | 99.83 | 4.07 | 99.66 | 5.80 |
| 22 | " | 0.64 | 99.94 | 2.38 | " | 4.13 | " | 5.86 |
| 24. | " | 0.70 | " | 2.44 | 99.82 | 4.18 | 99.65 | 5.92 |
| 26. | 99.99 | 0.76 | " | 2.50 | , | 4.24 | 99.64 | 5.98 |
| 28. | , | 0.81 | 99.93 | 2.56 | 99.81 | $4 \cdot 30$ | 99.63 | 6.04 |
| 30. | * | 0.87 | " | 2.62 | " | 4.36 | ، | 6.09 |
| 32 . | " | 0.93 | " | 2.67 | 99.80 | 4.42 | 99.62 | 6.15 |
| 34 . | * | 0.99 | " | 2.73 | " | 4.48 | " | 6.21 |
| 36. | " | 1.05 | 99.92 | 2.79 | 99.79 | $4 \cdot 53$ | 99.61 | 6.27 |
| $3^{8}$. | " | 1.11 | " | 2.85 | " | 4.59 | 99.60 | 6.33 |
| 40 ! | " | 1.16 | " | 2.91 | 99.78 | 4.65 | 99.59 | 6.38 |
| $42 \cdot$ | * | 1.22 | 99.91 | ${ }^{2} 2.97$ | " | 4.71 | " | 6.44 |
| 44 . . | 99.98 | 1.28 | " | 3.02 | 99.77 | 4.76 | 99.58 | 6.50 |
| 46 . . | " | I. 34 | 99.90 | 3.08 | " | 4.82 | 99.57 | 6.56 |
| $4^{8}$. | " | 1.40 | " | 3.14 | 99.76 | 4.88 | 99.56 | 6.61 |
| 50 . . | " | 1.45 | " | 3.20 | " | 4.94 | " | 6.67 |
| $52 \cdot$ | " | 1.51 | 99.89 | 3.26 | 99.75 | 4.99 | 99.55 | 6.73 |
| 54. | " | 1.57 | " | $3 \cdot 3{ }^{\circ}$ | 99.74 | 5.05 | 99.54 | 6.78 |
| $56 .$. | 99.97 | 1.63 | " | $3 \cdot 37$ | 6 | 5.11 | 99.53 | 6.84 |
| 58. | " | I. 69 | "99.88 | 3.43 | 99.73 | 5.17 | 99.52 | 6.90 |
| 60. | " | 1.74 | " | 3.49 | " | 5.23 | 99.51 | 6.96 |
| $c=0.75$ | 0.75 | 0.01 | 0.75 | 0.02 | 0.75 | 0.03 | 0.75 | 0.05 |
| $c=1.00$ | 1.00 | 0.01 | 1.00 | 0.03 | 1.00 | 0.04 | 1.00 | 0.06 |
| $c=1.25$ | 125 | 0.02 | 1.25 | 0.03 | 1.25 | 0.05 | 1.25 | 0.08 |

[^9]TABLE V.-Continued.
Horizontal Distances and Elevations from Stadia Readings.

| Minutes. | $4^{\circ}$ |  | $5^{\circ}$ |  | $6^{\circ}$ |  | $7^{\circ}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Hor. Dist. | Diff. Elev. | Hor. Dist. | Diff. <br> Elev. | Hor. Dist. | Diff. <br> Elev. | Her. Dist. | Diff. <br> Elev. |
| - . . | 99.51 | 6.96 | 99.24 | 8.68 | 98.91 | 10.40 | 98.51 | 12.10 |
| 2. |  | 7.02 | 99.23 | 8.74 | 98.90 | 10.45 | 98.50 | 12.15 |
| 4 | 99.50 | 7.07 | 99.22 | 8.80 | 98.88 | 10.51 | 98.48 | 12.21 |
| 6 | 99.49 | 7.13 | 99.21 | 8.85 | 98.87 | 10.57 | 98.47 | 1226 |
| 8 | 99.48 | 7.19 | 99.20 | 8.91 | 98.86 | 10.62 | 98.46 | 12.32 |
| 10 | 99.47 | 7.25 | 99.19 | 8.97 | 98.85 | 10.68 | 98.44 | 12.38 |
| 12 | 99.46 | 7.30 | 99.18 | 9.03 | 98.83 | 10.74 | 98.43 | 12.43 |
| 14 |  | 7.36 | 99.17 | 9.08 | 98.82 | 10.79 | 98.41 | 12.49 |
| 16 | 99.45 | 7.42 | 99.16 | 9.14 | 98.81 | 10.85 | 98.40 | 12.55 |
| 18 | 99.44 | 7.48 | 99.15 | 9.20 | 98.80 | 10.91 | 98.39 | 12.60 |
| 20 | 99.43 | 7.53 | 99.14 | 9.25 | 98.78 | 10.96 | 98.37 | 12.66 |
| 22 | 99.42 | 7.59 | 99.13 | 9.31 | 98.77 | 11.02 | 98.36 | 12.72 |
| 24 | 99.41 | 7.65 | 99.11 | 9.37 | 98.76 | 11.08 | 98.34 | 12.77 |
| $26^{*}$. | 9940 | 7.71 | 99.10 | 9.43 | 98.74 | I1.13 | 98.33 | 12.83 |
| 28 | 99.39 | 7.76 | 99.09 | 9.48 | 98.73 | 11.19 | 98.31 | 12.88 |
| 30 | 99.38 | 7.82 | 99.08 | 9.54 | 98.72 | 11.25 | 98.29 | 12.94 |
| 32 | 99.38 | 7.88 | 99.07 | 9.60 | 98.71 | 11.30 | 98.28 | 13.00 |
| 34 | 99.37 | 7.94 | 99.06 | 9.65 | 98.69 | 11.36 | 98.27 | 13.05 |
| 36 | 99.36 | 7.99 | 99.05 | 9.71 | 98.68 | 11.42 | 98.25 | 13.11 |
| 38 | 99.35 | 8.05 | 99.04 | 9.77 | 98.67 | 11.47 | 98.24 | 13.17 |
| 40 | 99.34 | 8.11 | 99.03 | 9.83 | 98.65 | 11. 53 | 98.22 | 13.22 |
| 42 | 99.33 | 8.17 | 99.01 | 9.88 | 98.64 | 11.59 | 98.20 | 13.28 |
| 44 | 99.32 | 8.22 | 9900 | 9.94 | 98.63 | 11.64 | 98.19 | 13.33 |
| $46^{\prime \prime}$. | 99.31 | 8.28 | 98.99 | 10.00 | 98.61. | 11.70 | 98.17 | 13.39 |
| 48 \%. | 99.30 | 8.34 | 98.98 | 10.05 | 98.60 | 11.76 | 98.16 | 13.45 |
| 50 | 99.29 | 8.40 | 98.97 | 10.11 | 98.58 | 11.81 | 98.14 | 13.50 |
| 52 | 99.28 | 8.45 | 98.96 | 10.17 | 98.57 | 11.87 | 98.13 | ${ }^{1} 3.56$ |
| 54 | 99.27 | 8.51 | 98.94 | 10.22 | 98.56 | 11.93 | 98.11 | ${ }^{13} .61$ |
| 56 | 99.26 | 8.57 | 98.93 | 10.28 | 98.54 | 11.98 | 98.10 | 13.67 |
| 58 | 99.25 | 8.63 | 98.92 | 10.34 | 98.53 | 12.94 | 98.08 | 13.73 |
| 60 | 99.24 | 8.68 | 98.91 | 10.40 | 98.51 | 12.10 | 98.06 | 13.78 |
| $c=0.75$ | 0.75 | 0.06 | 0.75 | 0.07 | 0.75 | 0.08 | 0.74 | 0.10 |
| $c=1.00$ | 1.00 | 0.08 | 0.99 | 0.09 | 0.99 | 0.11 | 0.99 | 0.13 |
| $c=1.25$ | 1.25 | 0.10 | 1. 24 | 0.11 | 1.24 | 0.14 | 1. 24 | 0.16 |

TABLE V.-Continued.
Horizontal Disíances and Elevations from Stadia Readings.

| Minutes. | $8^{\circ}$ |  | $9^{\circ}$ |  | $10^{\circ}$ |  | $11^{\circ}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Hor. <br> Dist. | Diff. <br> Elev. | Hor. <br> Dist. | Diff. <br> Elev. | Hor. Dist. | Diff. <br> Elev. | Hor. Dist. | Diff. <br> Elev. |
| $\bigcirc$ | 98.06 | 13.78 | $97 \cdot 55$ | I 5.45 | 96.98 | 17.10 | 96.36 | 18.73 |
| 2 | 98.05 | 13.84 | $97 \cdot 53$ | 15.51 | 96.96 | 17.16 | 96.34 | 18.78 |
| 4 | 98.03 | 13.89 | $97 \cdot 52$ | 15.56 | 96.94 | 17.21 | 96.32 | 18.84 |
| 6 | 98.01 | I 3.95 | $97 \cdot 50$ | 15.62 | 96.92 | 17.26 | 96.29 | 18.89 |
| 8 | 98.00 | 14.01 | 97.48 | I 5.67 | 96.90 | 17.32 | 96.27 | 18.95 |
| 10 | 97.98 | 14.06 | 97.46 | 15.73 | 96.88 | 17.37 | 96.25 | 19.00 |
| 12 | 97.97 | 14.12 | 97.44 | 15.78 | 96.86 | 17.43 | 96.23 | 19.05 |
| 14 | 97.95 | 14.17 | 97.43 | 15.84 | 96.84 | 17.48 | 96.21 | 19.11 |
| 16 | 97.93 | 14.23 | 9741 | 15.89 | 96.82 | 17.54 | 96.18 | 19.16 |
| 18 | 97.92 | 14.28 | 97.39 | 15.95 | 96.80 | 17.59 | 96.16 | 19.21 |
| 20 | 97.90 | 14.34 | $97 \cdot 37$ | 16.00 | 96.78 | 17.65 | 96.14 | 19.27 |
| 22 | 97.88 | 14.40 | -97.35 | 16.06 | 96.76 | 17.70 | 96.12 | 19.32 |
| 24 | 97.87 | 14.45 | $97 \cdot 33$ | 16.11 | 96.74 | 17.76 | 96.09 | 19.38 |
| 26 | 97.85 | 14.51 | 97.31 | 16.17 | 96.72 | 17.81 | 96.07 | 19.43 |
| 28 | 97.83 | 14.56 | 97.29 | 16.22 | 96.70 | 17.86 | 96.05 | 19.48 |
| 30 | 97.82 | 14.62 | 97.28 | . 16.28 | 96.68 | 17.92 | 96.03 | 19.54 |
| 32 | 97.80 | 14.67 | 97.26 | 16.33 | 96.66 | 17.97 | 96.00 | 19.59 |
| 34. | 97.78 | 14.73 | 97.24 | 16.39 | 96.64 | 18.03 | 95.98 | 19.64 |
| 36. | 97.76 | 14.79 | 97.22 | 16.44 | 96.62 | 18.08 | 95.96 | 19.70 |
| 38. | 97.75 | 14.84 | 97.20 | 16.50 | 96.60 | 18.14 | 95.93 | 19.75 |
| 40 | 97.73 | 14.90 | 97.18 | 16.55 | 96.57 | 18.19 | 95.91 | 19.80 |
| 42 | 97.71 | 14.95 | 97.16 | 16.61 | 96.55 | 18.24 | 95.89 | 19.86 |
| 44 | 97.69 | I 5.01 | 97.14 | 16.66 | 96.53 | 18.30 | 95.86 | 19.91 |
| 46. | 97.68 | 15.06 | 97.12 | 16.72 | 96.51 | 18.35 | 95.84 | 19.96 |
| 48. | 97.66 | 15.12 | 97.10 | 16.77 | 96.49 | 18.41 | 95.82 | 20.02 |
| 50 | 97.64 | 15.17 | 97.08 | 16.83 | 96.47 | 18.46 | 95.79 | 20.07 |
| 52. | 97.62 | 15.23 | 97.06 | 16.88 | 96.45 | 18.51 | 95.77 | 20.12 |
| 54. | 97.61 | 15.28 | 97.04 | 16.94 | 96.42 | 18.57 | 95.75 | 20.18 |
| 56. | $97 \cdot 59$ | 15.34 | 97.02 | 16.99 | 96.40 | 18.62 | 95.72 | 20.23 |
| 58 | 97.57 | 15.40 | 97.00 | 17.05 | 96.38 | 18.68 | 95.70 | 20.28 |
| 60 | $97 \cdot 55$ | 15.45 | 96.98 | 17.10 | 96.36 | 18.73 | 95.68 | 20.34 |
| $c=0.75$ | 0.74 | O.I I | 0.74 | 0.12 | 0.74 | 0.14 | 0.73 | 0.15 |
| $c=1.00$ | 0.99 | 0.15 | 0.99 | 0.16 | 0.98 | 0.18 | 0.98 | 0.20 |
| $c=1.25$ | 1.23 | 0.18 | 1.23 | 0.21 | 1.23 | 0.23 | 1.22 | 0.25 |

TABLE V.-Continued.
Horizontal Distances and Elevations from Stadia Readings.

| Minutes. | 120 |  | $13^{\circ}$ |  | $14{ }^{\circ}$ |  | $15^{\circ}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Hor. | Diff. Elev. | Hor. <br> Dist. | Diff. Elev. | Hor. Dist. | Diff. <br> Elev. | Hor. Dist. | Diff. <br> Elev. |
| - . | 95.68 | 20.34 | 94.94 | 21.92 | 94.15 | 23.47 | 93.30 | 25.00 |
| 2. | 95.65 | 20.39 | 94.91 | 21.97 | 94.12 | 23.52 | 93.27 | 25.05 |
| 4 . | 95.63 | 20.44 | 94.89 | 22.02 | 94.09 | 23.58 | 93.24 | 25.10 |
| 6 | 95.61 | 20.50 | 94.86 | 22.08 | 94.07 | 23.63 | 93.21 | 25.15 |
| 8 | 95.58 | 20.55 | 94.84 | 22.13 | 94.04 | 23.68 | 93.18 | 25.20 |
| 10 | 95.56 | 20.60 | 94.8I | 22.18 | 94.01 | 23.73 | 93.16 | 25.25 |
| 12 | 95.53 | 2066 | 94.79 | 22.23 | 93.98 | 23.78 | 93.r3 | 25.30 |
| 14 | 95.51 | 20.71 | 94.76 | 22.28 | 93.95 | 23.83 | 93.10 | 25.35 |
| 16 | 95.49 | 20.76 | 94.73 | 22.34 | 93.93 | 23.88 | 93.07 | 25.40 |
| 18 | 95.46 | 20.81 | 94.71 | 22.39 | 93.90 | 23.93 | 93.04 | 25.45 |
| 20 | $95 \cdot 44$ | 20.87 | 94.68 | 22.44 | 93.87 | 23.99 | 93.01 | 25.50 |
| 22 | 95.41 | 20.92 | 94.66 | 22.49 | 93.84 | 24.04 | 92.98 | 25.55 |
| 24 | $95 \cdot 39$ | 20.97 | 94.63 | 22.54 | 93.81 | 24.09 | 92.95 | 25.60 |
| 26 | 95.36 | 21.03 | 94.60 | 22.60 | 93.79 | 24.14 | 92.92 | 25.65 |
| 28 | $95 \cdot 34$ | 21.08 | 94.58 | 22.65 | 93.76 | 24.19 | 92.89 | 25.70 |
| 30 | $95 \cdot 32$ | 21.13 | 94.55 | 22.70 | 93.73 | 24.24 | 92.86 | 25.75 |
| 32 | 95.29 | 21.18 | 94.52 | 22.75 | 93.70 | 24.29 | 92.83 | 25.80 |
| 34 | 95.27 | 21.24 | 94.50 | 22.80 | 93.67 | 24.34 | 92.80 | 25.85 |
| 36 | 95.24 | 21.29 | 94.47 | 22.85 | 93.65 | 24.39 | 92.77 | 25.90 |
| 38 | 95.22 | 21.34 | 94.44 | 22.91 | 93.62 | 24.44 | 92.74 | 25.95 |
| 40 | 95.19 | 21.39 | 94.42 | 22.96 | 93.59 | 24.49 | 92.71 | 26.00 |
| 42 | 95.17 | 21.45 | 94.3) | 23.01 | 93.56 | 24.55 | 92.68 | 26.05 |
| 44 | 95.14 | 21.50 | 94.36 | 23.06 | 93.53 | 24.60 | 92.65 | 26.10 |
| 46 | 95.12 | 21.55 | $94 \cdot 34$ | 23.11 | 93.50 | 24.65 | 92.62 | 26.15 |
| 48 | 95.09 | 21.60 | 94.31 | 23.16 | 93.47 | 24.70 | 92.59 | 26.20 |
| 50 | 95.07 | 21.66 | 94.28 | 23.22 | 93.45 | 24.75 | 92.56 | 26.25 |
| 52 | 95.04 | 21.71 | 94.26 | 23.27 | 93.42 | 24.80 | 92.53 | 26.30 |
| 54 | 95.02 | 21.76 | 94.23 | 23.32 | 93.39 | 24.85 | 92.49 | 26.35 |
| 56 | 94.99 | 21.81 | 94.20 | 23.37 | 93.36 | 24.90 | 92.46 | 26.40 |
| 58 | 94.97 | 21.87 | 94.17 | 23.42 | 93.33 | 24.95 | 92.43 | 26.45 |
| 60 | 94.94 | 21.92 | 94.15 | 23.47 | 93.30 | 25.00 | 92.40 | 26.50 |
| $c=0.75$ | 0.73 | 0.16 | 0.73 | 0.17 | 0.73 | 0. 19 | 0.72 | 0.20 |
| $c=1.00$ | 0.98 | 0.22 | 0.97 | 0.23 | 0.97 | 0.25 | 0.96 | 0.27 |
| $c=1.25$ | 1.22 | 0.27 | 1.21 | 0.29 | 1.21 | 0.31 | 1.20 | 0.34 |

TABLE V.-Continued.
Horizontal Distances and Elevations from Stadia Readings.

| Minutes. | $16^{\circ}$ |  | $17^{\circ}$ |  | $18^{\circ}$ |  | $19^{\circ}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Hor. Dist. | Diff. <br> Elev. | Hor. Dist. | Diff. <br> Elev. | Hor. Dist. | Diff. <br> Elev. | Hor. Dist. | Diff. <br> Elev. |
| - | 92.40 | 26.50 | 9 I .45 | 27.96 | 90.45 | 29.39 | 89.40 | 30.78 |
| 2. | 92.37 | 26.55 | 9 I .42 | 28.01 | 90.42 | 29.44 | 89.36 | 30.83 |
| 4 | 92.34 | 26.59 | 91.39 | 28.06 | 90.38 | 29.48 | 89.33 | 30.87 |
| 6 | 92.31 | 26.64 | ${ }^{91} 35$ | 28.10 | 90.35 | 29.53 | 89.29 | 30.92 |
| 8 | 92.28 | 26.69 | 9 r .32 | 28.15 | 90.31 | 29.58 | 89.26 | 30.97 |
| 10 | 92.25 | 26.74 | 91.29 | 28.20 | 90.28 | 29.62 | 89.22 | 31.01 |
| 12 | 92.22 | 26.79 | 91.26 | 28.25 | 90.24 | 29.67 | 89.18 | 31.06 |
| 14 | 92.19 | 26.84 | 91.22 | 28.30 | 90.21 | 29.72 | 89.15 | 31.10 |
| 16. | 92.15 | 26.89 | 91.19 | 28.34 | 90.18 | 29.76 | 89.11 | 31.15 |
| 18 | 92.12 | 26.94 | 91.16 | 28.39 | 90.14 | 29.81 | 89.08 | 31.19 |
| 20 | 92.09 | 26.99 | 91.12 | 28.44 | 90.11 | 29.86 | 89.04 | 31.24 |
| 22 | 92.06 | 27.04 | 91.09 | 28.49 | 90.07 | 29.90 | 89.00 | 31.28 |
| 24 | 92.03 | 27.09 | 91.06 | 28.54 | 90.04 | 29.95 | 88.96 | 31.33 |
| 26 | 92.00 | 27.13 | 91.02 | 28.58 | 90.00 | 30.00 | 88.93 | 3 I .38 |
| 28 | 91.97 | 27.18 | 90.99 | 28.63 | 89.97 | 30.04 | 88.89 | 31.42 |
| 30 | 91.93 | 27.23 | 90.96 | 28.68 | 89.93 | 30.09 | 88.86 | 3 I .47 |
| 32 | 91.90 | 27.28 | 90.92 | 28.73 | 89.90 | 30.14 | 88.82 | 31.51 |
| 34 | 91.87 | 27.33 | 90.89 | 28.77 | 89.86 | 30.19 | 88.78 | 31.56 |
| 36 | 91.84 | 27.38 | 90.86 | 28.82 | 89.83 | 30.23 | 88.75 | 31.60 |
| 38. | 91.81 | 27.43 | 90.82 | 23.87 | 89.79 | 30.28 | 88.71 | 31.65 |
| 40 | 9 r .77 | 27.48 | 90.79 | 28.92 | 89.76 | 30.32 | 88.67 | 31.69 |
| 42 | 9 r .74 | 27.52 | 90.76 | 28.96 | 89.72 | 30.37 | 88.64 | 3 S .74 |
| 44 | 91.71 | 27.57 | 90.72 | 29.01 | 89.69 | 30.41 | 88.60 | 31.78 |
| 46. | $9 \mathrm{9r} .68$ | 27.62 | 90.69 | 29.06 | 89.65 | 30.46 | 88.56 | 31.83 |
| 48 | 91.65 | 27.67 | 90.66 | 29.11 | 89.61 | 30.51 | 88.53 | 31.87 |
| 50 | 91.61 | 27.72 | 90.62 | 29.15 | 89.58 | 30.55 | 88.49 | 31.92 |
| 52 | 91.58 | 27.77 | 90.59 | 29.20 | 89.54 | 30.60 | 88.45 | 3 F .96 |
| 54. | 91. 55 | 27.81 | 90.55 | 29.25 | 89.51 | 30.65 | 88.41 | 32.01 |
| 56. | 91.52 | 27.86 | 90.52 | 29.30 | 89.47 | 30.69 | 88.38 | 32.05 |
| 58. | 91.48 | 27.91 | 90.48 | 29.34 | 89.44 | 30.74 | 88.34 | 32.09 |
| 60 | 9 T .45 | 27.96 | 90.45 | 29.39 | 89.40 | 30.78 | 88.30 | 32.14 |
| $c=0.75$ | 0.72 | 0.21 | 0.72 | 0.23 | 0.71 | 0.24 | 0.71 | 0.25 |
| $c=1.00$ | 0.86 | 0.28 | 0.95 | 0.30 | 0.95 | 0.32 | 0.94 | 0.33 |
| $c=1.25$ | 1.20 | 0.35 | 1.19 | 0.38 | 1.19 | 0.40 | 1.18 | 0.42 |

TABLE V.-Continued.
Horizontal Distances and Elevations from Stadia Readings.

| Minutes. | $20^{\circ}$ |  | $21^{\circ}$ |  | $22^{\circ}$ |  | $23^{\circ}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Hor. <br> Dist. | Diff. Elev. | Hor. <br> Dist. | Diff. Elev. | $\begin{aligned} & \text { Hor. } \\ & \text { Dist. } \end{aligned}$ | Diff. <br> Elev. | Hor. Dist. | Diff. Elev. |
| $\bigcirc$ | 88.30 | 32.14 | 87.16 | 33.46 | 85.97 | 34.73 | 84.73 | 35.97 |
| 2. | 88.26 | 32.18 | 87.12 | 33.50 | 85.93 | 34.77 | 84.69 | 36.01 |
| 4. | 88.23 | 32.23 | 87.08 | 33.54 | 85.89 | 34.82 | 84.65 | 36.05 |
| 6 | 88.19 | 32.27 | 87.04 | 33.59 | 85.85 | 34.86 | 84.61 | 36.09 |
| 8 | 88.15 | 32.32 | 87.00 | 33.63 | 85.80 | 34.90 | 84.57 | 36.13 |
| 10 | 88.11 | 32.36 | 86.96 | 33.67 | 85.76 | 34.94 | 84.52 | 36.17 |
| 12 | 88.08 | 32.41 | 86.92 | 33.72 | 85.72 | 34.98 | 84.48 | 36.21 |
| 14 | 88.04 | 32.45 | 86.88 | 33.76 | 85.68 | 35.02 | 84.44 | 36.25 |
| 16 | 88.00 | 32.49 | 86.84 | 33.80 | 85.64 | 35.07 | 84.40 | 36.28 |
| 18 | 87.96 | 32.54 | 86.80 | 33.84 | 85.60 | 35.11 | 84.35 | 36.33 |
| 20 | 87.93 | 32.58 | 86.77 | 33.89 | 85.56 | 35.15 | 84.3 I | 36.37 |
| 22 | 87.89 | 32.63 | 86.73 | 33.93 | 85.52 | 35.19 | 84.27 | 36.41 |
| 24 | 87.85 | 32.67 | 86.69 | 33.97 | 85.48 | 35.23 | 84.23 | 36.45 |
| 26 | 87.81 | 32.72 | 86.65 | 34.01 | 85.44 | 35.27 | 84.18 | 36.49 |
| 28 | 87.77 | 32.76 | 86.61 | 34.06 | 8540 | 35.31 | 84.14 | 36.53 |
| 30 | 87.74 | 32.80 | 86.57 | 34.10 | 85.36 | $35 \cdot 36$ | 84.10 | 36.57 |
| 32 | 87.70 | 32.85 | 86.53 | 34.14 | 85.31 | 35.40 | 84.06 | 36.61 |
| 34 | 87.66 | 32.89 | 86.49 | 34.18 | 85.27 | $35 \cdot 44$ | 84.01 | 36.65 |
| 36 | 87.62 | 32.93 | 8645 | 34.23 | 85.23 | $35 \cdot 48$ | 83.97 | 36.69 |
| 38 . | 87.58 | 32.98 | 86.41 | 34.27 | 85.19 | 35.52 | 83.93 | 36.73 |
| 40 | 87.54 | 33.02 | 86.37 | $34 \cdot 3 \mathrm{I}$ | 85.15 | 35.56 | 83.89 | 36.77 |
| 42 | 87.51 | 33.07 | 86.33 | $34 \cdot 35$ | 85.11 | 35.60 | 83.84 | 36.80 |
| 44 | 87.47 | 33.11 | 86.29 | $34 \cdot 40$ | 85.07 | 35.64 | 83.80 | 36.84 |
| 46 | 87.43 | 33. 5 | 86.25 | 34.44 | 85.02 | 35.68 | 83.76 | 36.88 |
| 48 | 87.39 | 33.20 | 86.21 | 34.48 | 84.98 | 35:72 | 83.72 | 36.92 |
| 50 | 87.35 | 33.24 | 86.17 | 34.52 | 84.94 | 35.76 | 83.67 | 36.96 |
| 52 | 87.31 | 33.28 | 86.13 | 34.57 | 84.90 | 35.80 | 83.63 | 37.00 |
| 54 | 87.27 | $33 \cdot 33$ | 86.09 | 34.61 | 84.86 | 35.85 | 83.59 | 37.04 |
| 56 | 87.24 | 33.37 | 86.05 | 34.65 | 84.82 | 35.89 | 83.54 | 37.08 |
| 58 | 87.20 | 33.41 | 86.01 | 34.69 | 84.77 | 35.93 | 83.50 | 37.12 |
| 60 | 87.16 | 33.46 | 85.97 | 34.73 | 84.73 | 35.97 | 83.46 | 37.16 |
| $c=0.75$ | 0.70 | 0.26 | 0.70 | 0.27 | 0.69 | 0.29 | 0.69 | 0.30 |
| $c=1.00$ | 0.94 | 0.35 | 0.93 | 0.37 | 0.02 | 0.38 | 0.92 | 0.40 |
| $c=1.25$ | 1.17 | 0.44 | 1.16 | 0.46 | 1.15 | 0.48 | 1.15 | 0.50 |

## TABLE V.-Continued.

Horizontal Distances and Elevations from Stadia Readings.

| Minutes. | $24^{\circ}$ |  | $25^{\circ}$ |  | $26^{\circ}$ |  | $27^{\circ}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Hor. <br> Dist | Diff. <br> Elev. | Hor. <br> Dist. | Diff. Elev. | Hor. Dist. | $\begin{aligned} & \text { Diff. } \\ & \text { Elev } \end{aligned}$ | Hor. Dist. | Diff. Elev. |
| - | 83.46 | 37.16 | 82.14 | 38.30 | 80.78 | 39.40 | 79.39 | 40.45 |
| 2. | 83.41 | 37.20 | 82.09 | 38.34 | 80.74 | 39.44 | 79.34 | 40.49 |
| 4 | 83.37 | 37.23 | 82.05 | 38.38 | 80.69 | 39.47 | 79.30 | 40.52 |
| 6 | 83.33 | 37.27 | 82.01 | 38.41 | 80.65 | 39.51 | 79.25 | 40.55 |
| 8 | 83.28 | 37.31 | 81.96 | 38.45 | 80.60 | 39.54 | 79.20 | 40.59 |
| 10 | 83.24 | 37.35 | 81.92 | 38.49 | 80. 55 | 39.58 | 79.15 | 40.62 |
| 12 | 83.20 | 37.39 | 8 8 .87 | 38.53 | 80.51 | 39.61 | 79.11 | 40.66 |
| 14 | 83.15 | 37.43 | 8 I .83 | $3^{8.56}$ | 80.46 | 39.65 | 79.06 | 40.69 |
| 16 | 83.11 | 37.47 | 81.78 | 38.60 | 80.41 | 39.69 | 79.01 | 40.72 |
| 18 | 83.07 | 37.51 | 81.74 | 38.64 | 80.37 | 39.72 | 78.96 | 40.76 |
| 20 | 83.02 | 37.54 | 8 8 .69 | 38.67 | 80.32 | 39.76 | 78.92 | 40.79 |
| 22 | 82.98 | 37.58 | 8 r .65 | 38.71 | 80.28 | 39.79 | 78.87 | 40.82 |
| 24 | 82.93 | 37.62 | 81.60 | 38.75 | 80.23 | 39.83 | 78.82 | 40.86 |
| 26 | 82.89 | 37.66 | 81. 56 | 38.78 | 80.18 | 39.86 | 78.77 | 40.89 |
| 28 | 82.85 | 37.70 | 81.51 | 38.62 | 80.14 | 39.90 | 78.73 | 40.92 |
| 30 | 82.80 | 37.74 | 81.47 | 38.86 | S0.09 | 39.93 | 78.68 | 40.96 |
| 32 | 82.76 | 37.77 | 8 8 .42 | 38.89 | 80.04 | 39.97 | 78.63 | 40.99 |
| 34 | 82.72 | 37.81 | 8 I .38 | 38.93 | 80.00 | 40.00 | 78.58 | 41.02 |
| 36 | 82.67 | 37.85 | 81.33 | 38.97 | 79.95 | 40.04 | 78.54 | 41.06 |
| 38 | 82.63 | 37.89 | 8 r .28 | 39.00 | 79.90 | 40.07 | 78.49 | 41.09 |
| 40 | 82.58 | 37.93 | 81. 24 | 39.04 | 79.86 | 40.11 | 78.44 | 41.12 |
| 42 | 82.54 | 37.96 | 81.19 | 39.08 | 79.81 | 40.14 | 78.39 | 41.16 |
| 44 | 82.49 | 38.00 | 81.15 | 39.11 | 79.76 | 40.18 | 78.34 | 41.19 |
| 46 | 82.45 | 38.04 | 81.10 | 39.15 | 79.72 | 40.21 | 78.30 | 4 r .22 |
| 48 | 82.41 | 38.08 | 81.06 | 39.18 | 79.67 | 40.24 | 78.25 | 41.26 |
| 50 | 82.36 | 38.11 | 8 f .01 | 39.22 | 79.62 | 40.28 | 78.20 | 41.29 |
| 52. | 82.32 | 38.15 | 80.97 | 39.26 | 79.58 | 40.31 | 78.15 | 41.32 |
| 54 | 82.27 | 38.19 | 80.92 | 39.29 | 79.53 | 40.35 | 78.10 | 41.35 |
| 56. | 82.23 | 38.23 | 80.87 | 39.33 | 79.48 | 40.38 | 78.06 | 41.39 |
| 58 | 82.18 | 38.26 | 80.83 | 39.36 | 79.44 | 40.42 | 78.01 | 41.42 |
| 60 | 82.14 | 38.30 | 80.78 | 39.40 | 79.39 | 40.45 | 77.96 | 41.45 |
| $c=0.75$ | 0.68 | 0.31 | 0.68 | 0.32 | 0.67 | 0.33 | 0.66 | 0.35 |
| $c=1.00$ | 0.91 | 0.41 | 0.90 | 0.43 | 0.89 | 0.45 | 0.89 | 0.46 |
| $c=1.25$ | 1.14 | 0.52 | 1.13 | 0.54 | 1.12 | 0.56 | 1.11 | 0.58 |

TABLE V.-Continued.
Horizontal Distances and Elevations from Stadia Readings.

| Minutes. | $28^{\circ}$ |  | $29^{\circ}$ |  | $30^{\circ}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Hor. Dist: | Diff. Elev. | Hor. <br> Dist | Diff. <br> Elev. | Hor. Dist. | Diff. <br> Elev. |
| $\bigcirc$ | 77.96 | 41.45 | 76.50 | 42.40 | 75.00 | $43 \cdot 30$ |
| 2 | 77.91 | 41.48 | 76.45 | 42.43 | 74.95 | 43.33 |
| 4 | 77.86 | 41.52 | 76.40 | 42.46 | 74.90 | $43 \cdot 36$ |
| 6 | 77.81 | 41.55 | 76.35 | 42.49 | 74.85 | 43.39 |
| 8 | 77.77 | 41.58 | 76.30 | 42.53 | 74.80 | $43 \cdot 42$ |
| 10 | 77.72 | 41.61 | 76.25 | 42.56 | 74.75 | $43 \cdot 45$ |
| 12 | 77.67 | 41.65 | 76.20 | 42.59 | 74.70 | $43 \cdot 47$ |
| 14 | 77.62 | 4 t . 68 | 76.15 | 42.62 | 74.65 | 43.50 |
| 16 | 77.57 | 41.71 | 76.10 | 42.65 | 74.60 | 43.53 |
| 18 | 77.52 | 41.74 | 7 7.05 | 42.68 | 74.55 | 43.56 |
| 20 | 77.48 | 41.77 | 76.00 | 42.71 | 74.49 | 43.59 |
| 22 | 77.42 | 4 I .8 I | 75.95 | 42.74 | 74.44 | 43.62 |
| 24. | 77.38 | 41.84 | 75.90 | 42.77 | 74.39 | 43.65 |
| 26. | 77.33 | 41.87 | 75.85 | 42.80 | 74.34 | 43.67 |
| 28 | 77.28 | 41.90 | 75.80 | 42.83 | 74.29 | 43.70 |
| 30 | 77.23 | 41.93 | 75.75 | 42.86 | 74.24 | 43.73 |
| 32 - | 77.18 | 41.97 | 75.70 | 42.89 | 74.19 | 43.76 |
| 34 | 77.13 | 42.00 | 75.65 | 42.92 | 74.14 | 43.79 |
| 36. | 77.09 | 42.03 | 75.60 | 42.95 | 74.09 | 43.82 |
| 38 . | 77.04 | 42.06 | 75.55 | 42.98 | 74.04 | 43.84 |
| 40 | 76.99 | 42.09 | 75.50 | 43.01 | 73.99 | 43.87 |
| 42 | 76.94 | 42.12 | 75.45 | 43.04 | 73.93 | 43.90 |
| 44 | 76.89 | 42.15 | 75.40 | 43.07 | 73.88 | 43.93 |
| 46. | 76.84 | 42.19 | 75.35 | 43.10 | 73.83 | 43.95 |
| 48 | 76.79 | 42.22 | 75.30 | 43.13 | 73.78 | 43.98 |
| 50 | 76.74 | 42.25 | 75.25 | 43.16 | 73.73 | 44.01 |
| 52 | 76.69 | 42.28 | 75.20 | 43.18 | 73.68 | 44.04 |
| 54 | 76.64 | 42.31 | 75.15 | 43.21 | 73.63 | 44.07 |
| 56 | 76.59 | 42.34 | 75.10 | 43.24 | 73.58 | 44.09 |
| 58. | 76.55 | 42.37 | 75.05 | 43.27 | 73.52 | 44.12 |
| 60 | 76.50 | 42.40 | 75.00 | $43 \cdot 30$ | 73.47 | 44.15 |
| $c=0.75$ | 0.66 | 0.36 | 0.65 | 0.37 | 0.65 | 0.38 |
| $c=1.00$ | 0.88 | 0.48 | 0.87 | 0.49 | 0.86 | 0.51 |
| $c=1.25$ | 1.10 | 0.60 | 1.09 | 0.62 | 1.08 | 0.64 |

## TABLE VI.

Natural Sines and Cosines.

|  |  |  |  |  |  |  |  |  | $4{ }^{\circ}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Sine |  |  |  |  |  |  |  |  |  |  |
| 0 | . 000 | O | . 01745 |  |  |  | .05234 |  |  |  | 60 |
| 1 | . 000 |  |  |  | 03548 |  | . 05263 | . 9988 | - 703 | 4 | 59 |
| 2 | $\left\lvert\, \begin{gathered} .00058 \\ 00087 \end{gathered}\right.$ | One | . 018 | . 999988 | . 03548 | . 9993 | .05292 | . 9986 | . 07034 | 99752 | 58 |
|  | . 00116 | One | . 018 | . 99 | . 03606 | . 99935 | . 05350 | . 998 | . 07092 |  | 5 |
|  | . 00145 | One. | . 01891 | . 999 | . 0363 | . 999 | . 05379 | . 99855 | 07121 | 99746 | 55 |
| 6 | . 00175 | One | . 01920 | . 99982 | . 03664 | . 99 | . 05408 | . 998 | . 07150 | 99744 | 54 |
| 7 | . 00204 | On | . 019 | . 999881 | . 036 | . 999332 | . 05 | 99852 | . 07179 | 99 | 53 |
| 8 | .002 | One | . 019 | . 99980 | . 037 | . 99931 | . 05 | . 99851 | . 07208 | 99 | 52 |
| 10 | . 0 |  |  |  |  |  |  |  |  |  | 50 |
| 11 |  |  |  | . 99979 |  | . 99927 |  |  | 5 |  | 49 |
| 12 | . 003 | . 999 |  |  |  |  |  |  |  |  | 48 |
| 13 | . 003 | . 99999 | . 021 | . 99977 | . 038 | . 99 | . 056 | . 998 | . 07353 | 997 | 47 |
| 14 | . 004 | . 99999 | . 021 | . 99977 | . 0389 | . 999 | . 056 | . 998 | . | 997 | 46 |
| 10 | . 00 | . 99999 | . 02181 | . 999976 | . 0392 | . 999 | . 056 | . 998 | 07411 |  | 45 |
| 16 | . 00 | . 99999 | . 02 | . 99976 | . 039 | . 999 | . 05 |  | . 07440 | 99 | 44 |
| 17 | . 00 | . 99999 | . 02 | . 999 | . 039 | . 9992 | . 05 | . 99 | . 07469 | 99 | 43 |
| 18 | . 005 | . 99999 | . 02269 | . 999 | . 40 | . 99919 | . 05 | . 99 | . 0 T 4 | 99719 | 1 |
| 19 | . 005 | . 9999 |  | . 999 | . 40 | . 99918 |  |  | . 075 | 99716 | 41 |
| 20 |  |  |  |  |  |  |  |  |  |  | 40 |
|  | . 0 |  |  |  | . 041 |  | . 05344 | . 99 | . 0 |  | 39 |
| 21 |  | . 9999 |  | . 999 | . 041 | . 99915 | . 05 | . 99 | . 076 |  | 38 |
| 23 | . 006 | . 9999 | . 02 | . 99971 | . 0415 | . 99913 | . 05 |  | . 176 |  | 37 |
| 2425 | . 0069 | . 99998 | . 02 | . 99970 | . 041 | . 99912 | . 05 | . 99 | . 07672 | 997 | 36 |
|  | . 007 | .9999\% | . 02472 | . 999 | 42 | . 99911 | . 05960 | . 99822 | . 07701 | 997 | 35 |
| 26 | . 007 | . 99997 |  | . 9996 | . 0424 | . 99910 | . 05 | . 99821 | . 07730 | 99 | 34 |
| ${ }_{27}^{20}$ | . 00785 | . 9999 | . 02 | . 9996 | 042 | . 99909 | . 060 |  | . 07759 | .99699 |  |
| $\begin{aligned} & 27 \\ & 28 \end{aligned}$ | 00814 | . 99997 |  | . 999 |  |  |  | . 99817 |  |  |  |
| $\begin{aligned} & 28 \\ & 29 \end{aligned}$ | . 00844 | . 99996 |  | . 999 |  | . 99 |  |  | . 07817 |  | 31 |
| 30 |  |  |  |  |  |  |  |  |  |  | 30 |
| 31 | . 0 |  | . 0 | . 99 | . 04391 | . 99 | . 06134 |  | . 0 |  | 29 |
| 213331 | . 009 | . 99 | . 026 | . 9999 | . 04 | . 99902 | . 061 |  | . 079 |  | 28 |
|  | . 00960 | . 99995 | . 027 | . 99963 | . 044 | . 99901 |  | . 99808 | . 079 |  | 27 |
|  | . 00989 | . 9999 | . 027 |  |  | . 99900 |  |  |  |  | 26 |
| 34 35 35 | . 01018 | . 99 |  | . 999 |  |  | . 062 |  |  |  | 25 |
| 36 | . 01047 | . 9939 | . 02 | . 99961 | . 045 | . 998 | . 06 |  | . 08020 |  | 24 |
| 373838 |  | . 99 |  | . 99960 | . 045 |  |  |  |  |  | 23 |
|  | . 01105 |  |  | . 999 |  |  |  | . 997 |  |  | 22 |
| 38 39 | . 011134 | . 99994 | . $028 \%$ | . 99959 | - | . 998 |  |  | . 08107 |  | 2 |
| 40 | . 0 |  |  |  |  | . 9 |  |  | . 08136 |  | 20 |
| 41 |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | . 99956 | . 047 | . 998 |  | . 997 | . 08194 |  | 18 |
| $43$ | . 01251 | . 99992 |  |  |  |  |  |  | . 08223 |  | 17 |
| $\begin{aligned} & 43 \\ & 44 \end{aligned}$ | . 01280 | . 99992 | . 030 | . 999 | . 047 |  | . 065 | . 997 | .0822 | . 99 | 16 |
| $\begin{aligned} & 44 \\ & 45 \end{aligned}$ | . 01309 | . 99991 | . 0305 | . 999 | . 047 |  |  |  | . 08281 |  | 1 |
| $\begin{aligned} & 45 \\ & 46 \end{aligned}$ | . 01338 | . 999991 | . 0301 |  |  |  |  |  | . 08310 | -9 | 14 |
| 47 | . 01367 | . 99991 | . 03112 | . 99 | . 048 | . 99 |  |  | . 083 | .996 | 13 |
| $\begin{aligned} & 47 \\ & 48 \\ & 48 \end{aligned}$ | . 01396 | . 99990 | . 03141 | . 99951 | . 048 |  |  | . 997 | . 0838 | 9964 | 1 |
| $\begin{aligned} & 48 \\ & 49 \end{aligned}$ |  | 300 |  | -.3950 |  |  |  | . 2378 | . 08397 | 996 | 11 |
| 50 |  |  |  |  |  |  |  |  |  |  | 10 |
| 51 | . 01483 | . 999 | .032 |  | O4 |  |  | . 99774 | . 084 |  |  |
| 53 | . 01 | . 999 |  | . 9994 | . 05001 | . 998 | . 067 | . 99772 | . 08484 | . 996 |  |
|  |  | . 9998 | . 0328 | . 999 | . 0503 | . 998 | . 0 | . 99 | . 08513 | 99 |  |
| $\begin{aligned} & 53 \\ & 54 \end{aligned}$ | . 01571 | . 393 | . 03316 | . 99945 | . 050 | . 9987 | . 068 | . 99768 | . 08542 | 996 |  |
| 55 | . 1 | . 9996 | . 03345 | . 99944 | . 050 | . 998 |  | . 99 | . 085 | . 996 | 5 |
|  | . 01629 | . 999 | . 03374 | . 999 | . 051 |  | . 068 | . 99764 | . | 996 |  |
| 56 58 | . | . 999 | . 03403 | . 99942 | . 0514 | . 998 | . 0688 | . 99762 | . 0862 | . 996 |  |
| 58 | . 0 | . 99 | . 03432 | . 999 | . 05 | . 998 | . 0691 |  | . 0865 | 99 |  |
| 59 <br> 60 | . 01716 |  |  | . 99940 | - | . 998 |  | 997 |  | 992 | 1 |
|  |  |  |  | . 99939 |  |  |  |  |  | . 99619 | 0 |
|  | C |  |  |  |  |  |  |  |  |  |  |
|  |  | $89^{\circ}$ | $88^{\circ}$ |  | $87^{\circ}$ |  | $86^{\circ}$ |  | $85^{\circ}$ |  |  |

## TABLE VI.-Continued.

## Natural Sines and Cosines.



TABLE VI. - Continued.
Natural Sines and Cosines.

|  |  |  | 11 |  |  |  | 13 |  |  |  | , |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Cosin | Sine | Cosin |  | Cosin | Sine | Cosin | Sine | Cosin |  |
|  | 1730 | . 98481 | . 19081 | . | 20 | . 97815 | 2 |  | 92 | 30 | 0 |
|  | 173 | . 98476 | . 19109 |  | 20 | . 978 |  | . 97430 | 220 | 97023 | 59 |
|  | . 17422 |  | 19138 | . 88 | . 20848 | . 97803 | . 222552 | . 97424 | 24249 | 97015 | 58 |
|  | 17451 | . 98466 | . 19167 | . 98146 | . 20877 | . 97797 | . 22580 | . 97417 | 24277 | 8 | 5 |
|  | .17479 | 98461 | . 19195 | . 98140 | . 20905 | . 977791 | . 22608 | . 97411 | 24305 | $9 \% 001$ | 56 |
|  | 17508 | . 98455 | . 19224 | . 98135 | . 209 | . 9777 | . 222637 | . 97404 | . 24333 | 96994 | 55 |
|  | 17537 | ${ }^{98145}$ | . 19252 | . 98129 | 20962 | . 9777 | . 22665 | . 773 | . 24362 |  | 54 |
|  | . 175 |  | . 192881 | . 98124 | . 21019 |  |  | . 97391 | .243918 | 96980 | 53 |
|  |  |  | . 19338 | . 98112 | 210 | . 97760 | . 227 | . 97378 | . 24446 |  | 51 |
| 10 | . 1 |  | . 19366 |  | 21076 | . 97754 | . 2 |  |  |  | 5 |
| 11 | . 17680 | . 98 |  | . 9810 |  |  | . 228 | . 97365 | . 24503 | . 96952 | 49 |
| 12 | . 17708 | . 984 | . 19 |  |  |  | . 228 |  | . 24531 |  | 48 |
| 13 | . 17737 | . 9841 | . 19 | . 9809 | . 21161 | . 97735 | . 228 | . 97 | 24559 |  | 47 |
| 14 | . 17766 | . 98409 | . 1948 | . 98084 | . 21 | . 97729 | . 22 | . 973 | . 24587 |  | 46 |
| 15 | . 17794 | . 98404 | . 19509 | . 98079 | . 212 | . 977 | . 22920 | . 973 | . 24615 | 96 | 45 |
| 16 | . 178 | . 98399 |  | . 980 |  | . 97 | . 22948 | . 973 | . 24644 |  | 4 |
| 17 | . 17850 | . 98394 | . 195 | . 98 | . 21 | . 97 | . 2297 | . 973 | . 24672 | . 96909 | 43 |
| 18 | . 17880 | . 9838 | . 195 | . 98 | . 21 | . 97 | . 2330 | . 973 | . 2470 | 96902 | 42 |
| 19 | . 17909 | . 98383 |  | . 98056 |  | . 97 | . 2303 | . 97311 |  |  | 41 |
| 20 | . 17 | . 983 |  |  |  |  |  |  |  |  | 40 |
| 21 | . 17966 | .983 | . 196 | . 980 |  | . 9 | . 23090 | . 97 | . 21784 | . 96880 | 39 |
|  | . 17995 | . 983 | . 197 | . 980 | . 21 | . 97 | . 23118 |  |  |  | 38 |
| 23 |  | . 9838 |  | . 93033 | . 214 | . 9767 | . 23 |  | . 24841 |  | 37 |
| 24 | . 18052 | . 9835 | . 197 | . 98027 | . 214 | . 97667 | . 231 |  | . 24869 |  | 36 |
| 25 | . 18081 | . 9835 | . 197 | . 98021 | . 21502 | . 97661 | . 2320 | . 97 | 24897 | 968 | 35 |
| 25 | . 18109 | . 98347 |  | . 98316 | . 21530 |  | . 2323 | . 9726 | . 24925 |  | 34 |
| 27 | . 1813 | . 98341 | . 19851 | . 93510 | . 215 | . 976 | . 23260 | . 9725 | . 21 |  | 33 |
| 28 | . 1816 | . 9833 | . 19880 | . 93004 | . 215 | . 9761 | . 23288 | . 972 | . 2498 | 96 | 32 |
| 29 | . 181 | . 98331 | . 1 | . 97993 | . 21616 |  | . 23316 | . 97244 | 2501 | .96822 |  |
| 30 | . 18 | . 9 |  |  |  |  | . 23 |  |  |  | 30 |
| 31 | . 182 | . 98 | . 199 | . 979 | . 216 | . $9 \%$ | . 233 | . 97230 | . 25066 |  | 29 |
| 82 | . 1828 | . 983 |  | . 9 T9 | . 217 | . 97 | . 2340 |  |  |  | 23 |
| 33 |  | . 98310 | . 2002 | . 97975 | . 217 | . 970 | . 234 |  | . 25122 |  | 27 |
| 34 | . 18338 | . 93304 | . 20051 | . 97963 | . 217 | . 97604 | . 23458 | . 97210 | . 25151 | 967 | 25 |
| 35 | . 18 | . 932 | . 200 | . 9796 | . 2178 | . 975 |  |  | . 25179 | . 96 | 25 |
| 36 |  | .9320 | . 2010 | 935 | . 21814 |  | . 23514 | . 97196 |  |  | 24 |
| 37 | . 1812 | . 9828 | . 20130 | . 9795 | . 21813 | . 975 | . 23542 | . 97189 | . 25235 |  | 23 |
| 38 |  |  |  |  | . 218 |  | . 2357 | . 97182 | . 25263 |  | 22 |
| 39 |  |  | . 2 |  | 218 |  | . 235 | . 97176 | 25291 |  | 21 |
| 40 | . 18 |  |  |  |  |  |  |  |  |  | 20 |
| 41 | . 185 | . 9826 | . 202 |  | . 219 |  |  |  | . 25348 |  | 19 |
| $\leq 2$ | . 18567 | . 9336 | . 20279 | . 9792 | . 2198 | . 9755 | . 2368 | . 97155 | . 25376 | . 967 | 18 |
|  | . 18595 |  | . 20307 | . 97916 | . 22013 | . 97547 | . 2371 | . 97148 | . 25404 | . 96719 | 17 |
| 44 | . 18624 | .98250 | . 20336 | . 97310 | . 22011 | . 97541 |  | . 97141 | . 25432 | . 96 | 15 |
| 45 | . 18652 | . 98245 | . 2036 | . 9730 | . 22070 | . 9 т534 | . 2376 | . 97134 | . 2546 | . 96 | 15 |
| 46 | . 18681 | . 93240 | . 20393 | . 97899 | 2098 | . 975 | . 237 | . 97127 | . 25488 |  | 14 |
|  | . 18710 | . 938231 | . 2042 | . 97893 | 2212 | . $9 \% 2$ | . 238 | . 97120 | . 25516 |  | 13 |
| 43 | . 1873 |  | . 2045 | . 9788 | . 2215 | . 97515 | . 2385 | . 97113 | . 25545 |  | 12 |
| 43 | . 1876 | .98223 | . 2047 | . 97881 | . 2218 |  | . 23882 | .9r106 | . 25573 |  | 1 |
| 50 |  |  |  |  |  |  |  |  |  |  | 10 |
| 51 | . 1882 | . 9821 |  |  |  |  | . 23 |  | . 25629 |  |  |
| 52 | . 18852 | .9820 | . 20563 | . 97863 | .2226 | . 9748 | . 2396 | . 97086 | . 256 |  |  |
|  | . 18881 | . 98201 | . 20592 | . 9785 | . 2229 | . 974 | . 2399 | . 97079 | . 2568 |  | 7 |
|  | - 1810 | . 98190 | -2020 | . 97851 | . 2232 | . 97476 | . 2402 | . 9707 | 25713 |  |  |
| 55 | . 18938 | . 98190 | . 20649 | . 97845 | . 2235 | . 97470 | . 24051 | . 97065 | 25741 |  |  |
|  | . 18967 | . 98185 | . 20677 | . 9783 | . 2238 |  | . 24079 | . 970 | 25769 | . | 4 |
|  | . 18999 | 98179 | . 20706 |  | . 2241 | . 9745 | . 241 | . 9705 | 25798 | . | 3 |
| 58 | . 19024 | . 98174 | . 2073 | . 9782 | . 2243 | . 97450 | . 2413 | . 970 | . 25826 |  | 2 |
| 59 | . 19052 | . 98168 | . 20763 | . 97821 | . 2246 | . 974 | . 24164 | . 97 | . 25854 | . 9660 | 1 |
| 60 | 81 |  |  |  |  |  |  |  |  |  | 0 |
|  | Cosin | Sine |  | Sine |  | Sin |  |  | Cosin | Sin |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

TABLE VI.-Continued.
Natural Sines and Cosines.

|  |  |  | $16^{\circ}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Sine |  |  |
| 0 | . 258 |  | - 2 | . 961 | . 292 | 95 | 2 | . 95106 | 32557 | 2 | 60 |
| 2 | . 25910 | 96 | . 27 | . 96 | . 292 | 95 | . 30929 | . 95097 | 34 | . 94542 | 59 |
|  | . 25938 | . 96578 | . 27620 | . 966110 | . 292938 | . 95613 | . 30957 | . 95088 | ${ }^{3} 32639$ | . 94533 | 58 |
| 4 | . 25994 | . 965 | . 27676 | . 96094 | . 29348 | . 95596 | . 31012 | . 95070 | . $3266{ }^{7}$ | 94 |  |
| 5 | . 26022 | . 96555 | . 2770 | . 9608 | . 29376 | . 95588 | . 31040 | . 95061 | . 32694 | 94504 | 55 |
|  | . 26050 | . 96547 | . 27731 | . 96078 | . 29404 | . 955579 | . 31068 | . 95052 | . 32722 | 94495 | 54 |
| 6 | . 26079 | . 96540 | . 27759 | -96070 | . 29432 | . $955 \% 1$ | . 31095 | . 95043 | . 32749 | 55 | 53 |
| $\begin{aligned} & 7 \\ & 8 \end{aligned}$ | . 26107 | 965 | . 2 | . 960 | . 294 | . 955562 | . 31123 | . 95033 | . 32777 | . 94476 | 52 |
| $\begin{aligned} & 8 \\ & 9 \end{aligned}$ | . 2613163 | 96:5 | . 27 | . 966 | . 294857 |  | . 311151 |  | . 322832 |  | 51 |
| 10 | . 2019 |  |  |  | . 29543 | . 95538 | . 31 |  | . 32859 |  | 49 |
|  | . 26219 | 965 | . 27899 | . 96029 | . 29571 | . 955 | . 31 | . 94997 | . 32887 |  | 48 |
| 12 | . 26247 | . $96 \% 94$ | . 2 \% | . 96021 | . 29599 | . 95519 | . 31261 | 94988 | . 32914 | 94 | 47 |
|  | . 26275 | $9648{ }^{\text {b }}$ | . 279 | . 96013 | . 296 | . 95511 | . | 94979 | . 32942 |  | 46 |
| 1 | . 26303 | . 96479 | . 27983 | . 96005 | . 29654 | . 95502 | . 31316 | . 94970 | . 32969 | 94409 | 45 |
| 16 | . 26331 | . 96471 | . 2801 | . 95997 | . 29682 | . 95493 | . 31344 | . 94961 | . 32997 | 94399 | 44 |
| 17 | . 26359 | . 9646 | . 28039 | . 95989 | . 29710 | . 9548 | . 31 | . 94952 | . 33024 |  | 43 |
| $18$ | . 26387 | . 96456 | . 28067 | . 95981 | . 27737 | . 95476 | . 31399 | . 94943 | . 33051 | 30 | 42 |
| 19 | . 26415 | . 9644 | . 2809 | . 95972 |  | . 95467 |  |  | . 33079 | 94370 | 41 |
| 20 | . 26443 | . 96440 | 8123 |  |  |  |  |  |  |  | 40 |
| 21 | . 26471 | . 964 | . 28150 | . 959 | . 298 | . 95450 | . 31 | 94915 | . 33134 |  | 39 |
|  | . 26500 | . 9642 | . 281 | . 959 | . 298 | . 95441 | . 31 | 94906 | . 33161 |  | 38 |
| 23 | . 26528 | . 96417 | . 28206 | . 95940 | . 29876 | . 95433 | . 31537 | . 94897 | . 33189 |  |  |
| 24 | . 26556 | . 96410 | . 28834 | . 05931 | . 29904 | . 95424 | . 315 | . 94888 | . 33216 |  |  |
|  | . 26584 | . 96402 | . 28262 | . 95923 | . 299 | . 05415 | . 31 | . 948 | . 33244 | 94313 | 35 |
| 26 | . 26612 | . 9639 | . 28290 | . 959 | . 299 | . 954 | . 31620 | . 94869 | . 33271 | . 94303 | 34 |
|  | . 26640 | . 96386 | . 28318 | . 959 | . 29987 | . 953 | . 31648 | . 94860 | . 33238 | . 94 | 33 |
| 28 | . 26668 | . 9637 | . 23346 | . 958 | . 30015 | . 953 |  | . 94851 |  |  |  |
| 29 | . 26696 | . 96371 | . 28374 | . 958 | . 30043 | . 95380 | . 31 | . 94842 | 35 | 94274 | 31 |
| 30 | . 26724 | . 96 | 8402 |  |  |  |  |  |  |  |  |
| 313232 | . 26752 | . 96 | . 28429 | . 95 | . 300 |  | . 31 | . 94823 | . 33408 | 94254 | 29 |
|  | . 26780 | . 96347 | . 28457 | . 958 | . 30126 | . 95354 | . 31 | . 94814 | . 33436 | . 94245 |  |
| 32 | . 26808 | . 963 | . 28 | . 9585 | . 30154 | . 953 | . 31813 | . 94805 |  |  | 27 |
| 33 | . 26836 | . 96332 | . 23513 | . 958 | . 30182 |  | . 31841 |  | . 33490 | . 94 | 26 |
| 34 | . 26864 | . 96334 | . 28541 | . 95841 | . 30209 | . 95338 | . 318 | . 94786 | . 33518 | . 94215 | 25 |
| 35 36 | . 26892 | . 96316 | . 28569 | . 958 | . 30237 | . 95319 | . 818 | . 94777 | . 33545 | 94206 | 24 |
| 36 | . 26920 | . 96308 | . 28597 | . 95824 | . 302 | . 95310 |  |  | . 33573 | . 94196 | 23 |
| $38$ | . 26948 | . 96301 |  | . 958816 |  | . 95301 | . 31951 |  | 33600 |  | 2 |
| 40 | ${ }^{.26976}$ |  |  | . 958 |  | . 95293 |  | . 94749 | 327 | 94176 | 21 |
| 41 | . 2 |  |  |  |  |  |  |  |  |  | 19 |
| 42 | . 27060 | . 962 | . 28 | . 957 | . 304 | . 95 | . 3206 | . $94 \sim 21$ | . 33710 | 94147 | 18 |
| 43 | . 27088 | . 96261 | . 28764 | . 9577 | . 30431 | . 95257 | . 3208 | . 94712 | 38737 |  | 17 |
| 4145 | . 27116 | . 96253 | . 28792 | . 95766 | . 30459 | . 95248 | . 32116 | . 94702 | . 33764 | . 94127 | 16 |
|  | . 27144 | . 962 | . 28820 | . 95757 | . 30486 | . 95240 | . 32144 | . 94693 | . 33192 | . 94118 | 15 |
| 45 | . $271 \% 2$ | . 9623 | . 28847 | . 95749 | . 30514 | . 952 | . 32171 | . 94684 | . 33819 |  | 14 |
| 46 | . 27200 | . 96330 | . 28875 | . 95740 | . 30542 | . 95222 | . 32199 | . 94674 | . 33846 | 94098 | 13 |
| $48$ | . 27228 | . 962 | . 28903 | . 95732 | . 30570 | . 95213 | . 3222 | . 94665 | . 33874 | . 94 | 12 |
| $49$ | . 27256 | 96214 | 28931 | . 95724 | . 30597 | . 95204 | . 32254 | . 94656 | 83901 | 94018 | 11 |
| 50 | . $2 \tau 284$ |  | 28959 | . 95715 |  |  | . 3228 |  | 33929 |  | 10 |
| 51 | . 27312 | . 96198 | . 28987 | . 9570 | . 30653 | . 95186 | . 32309 | . 94637 | . 33956 | 94058 |  |
| 52 | . 27340 | . 96190 | . 29015 | . 9563 | . 30680 | . 95177 | . 2233 | . 94627 | . 33983 | 94049 |  |
| 5 | . 27368 | . 96182 | . 29042 | . 95690 | . 30708 | 95168 | . 32364 | . 94618 | . 34011 | . 9403 | 7 |
| 54 | . 27396 | . 96174 | . 2907 | . 9568 | . 30736 | . 95159 | . 32392 | . 94609 | . 34038 | 94029 |  |
| 55 | . 27424 | . 9616 | . 29098 | . 9567 | . 30763 | . 95150 | . 32419 | . 94599 | . 34065 | . 94019 |  |
| 56 | . 27452 | . 96158 | . 29126 | . 9566 | .30791 | . 95142 | . 32447 | . 94590 | . 34093 | 94009 | 4 |
| 5 | . 27480 | . 96150 | . 2915 | . 95656 | . 30819 | 95133 | . 32474 | . 93580 | . 34120 | , | 3 |
| 58 | . 27508 | . 96142 | . 29182 | . 95647 | . 30846 | 95124 | . 32502 | . 94571 | 34147 | 939 | 2 |
| 59 | . 27536 | . 96 | 09 | . 9563 | . 30874 | . 9511 | . 32529 | . 945 | 34175 | 93979 | 1 |
| 60 |  |  |  |  | . 30902 |  | 3 |  | 02 | . 93969 | 0 |
|  | Cosin | Sine | Cosin | Sine | Cosin | Sine | Cosin | Si | Cosin | Sine |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

TABLE VI.-Continued.
Natural Sines and Cosines.


TABLE VI.-Continued.
Natural Sines and Cosines.

|  | $25^{\circ}$ |  |  | $26^{\circ}$ |  |  |  | $28^{\circ}$ |  | $29^{\circ}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Sine | Cos | Sine | Cos | Sin | Cosin | Sin | Co | Sine | Cosin |  |
|  |  | . 42262 | . 90631 | . 43837 | . 898879 | . 45399 | . 89101 | . 46947 | . 8 | . 48481 | 2 | 60 |
|  |  | . 4228 | . 90618 | . 43863 | . 89867 | . 454 | . 89087 | . 46973 | . 88281 | . 48506 | 87448 | 59 |
|  |  | . 42315 | . 90606 | . 43889 | . 898854 | . 4545477 | . 893074 | . 469992 | . 882687 | . 48532 | 87434 87420 | 58 |
| 4 |  | . 423367 | . 90582 | . 43942 | . 89888 | . 45503 | . 89048 | . 47050 | . 88240 | . 48583 | 87406 | 56 |
|  |  | . 42394 | . 90569 | . 43968 | . 89816 | . 45529 | . 89035 | . 47076 | . 88226 | . 48608 | 87391 | 55 |
| 6 |  | . 42420 | . 90557 | . 43994 | . 89803 | . 45554 | . 89021 | . 47101 | . 88213 | . 48634 | . 87377 | 54 |
|  |  | . 42446 | . 90545 | . 44020 | . 89790 | . 45550 | . 89008 | . 47127 | . 88199 | . 48659 | . 87363 | 53 |
|  |  | . 42473 | . 90532 | . 44046 | . 89777 | . 45606 | . 889985 | . 47153 | . 888185 | . 486810 | . 873349 | 52 |
| 10 |  | . 42425 | . 90550 | . $440 \% 2$ | .89764 | . 4563658 | . 8889818 | . 4777204 | .88172 | $\begin{array}{\|l\|} \hline .48710 \\ .48735 \end{array}$ | $.87335$ | 51 |
| 11 |  | . 42552 | . 90 | . 44124 | . 89739 | . 45684 | . 88955 | . 47229 | . 88144 | . 48761 | . 87306 | 4.) |
| 2 |  | . 42578 | . 90483 | . 44151 | . 89726 | . 45710 | . 88942 | . 47255 | . 88130 | . 48786 | 87292 | 48 |
| 13 |  | . 42604 | . 90470 | . 44177 | . 89713 | .45\%36 | . 88928 | . 47281 | . 88117 | . 48811 | . 87278 | 47 |
| 4 |  | . 42631 | . 90458 | . 44203 | . 89700 | . 45762 | . 88915 | . 47306 | . 88103 | . 48837 | . 87264 | 46 |
| 15 |  | . 42657 | . 90446 | . 44229 | . 89687 | . 45787 | . 88902 | . 47332 | . 88089 | . 48862 | . 87250 | 45 |
| 16 |  | . 42683 | . 90433 | . 44255 | . 89674 | . 45813 | . 88888 | . 47358 | . 88075 | . 48888 | . 87235 | 44 |
| 17 |  | . 4270 | . 90421 | . 44281 | . 89662 | . 45839 | .88875 | . 47383 | . 88062 | . 48913 | . 87221 | 43 |
| 18 |  | . 42736 | . 90408 | . 44307 | . 89649 | . 45865 | . 88882 | . 47409 | . 88048 | . 48938 | . 87207 | 42 |
| 19 |  | . 4276 | . 90393 | . 44333 | . 89636 | . 45891 | . 88848 | . 47434 | . 88034 | . 48964 | . 87193 | 41 |
| 20 |  | .4878 | . 90383 | . 44359 | . 89623 | . 45917 | . 88835 | . 47460 | . 88020 | . 48989 | 871 | 40 |
| 21 |  | . 42815 | . 90371 | . 41385 | . 89610 | . 45942 | . 88822 | . 47486 | . 88006 | . 49014 | . 87164 | 39 |
| 22 |  | . 42841 | . 90358 | . 44411 | . 89597 | . 45968 | . 8888 | . 47511 | . 87993 | . 49040 | . 87150 | 38 |
| 3 |  | . 42867 | . 90346 | . 44437 | . 89584 | . 45994 | . 88795 | . 47537 | . 87979 | . 49065 | . 87136 | 37 |
| 24 |  | . 42894 | . 90334 | . 44464 | .89571 | . 46020 | . 88782 | . 47562 | . 87965 | . 49090 | . 87121 | 36 |
| 25 |  | . 42920 | . 90321 | . 44490 | . 89558 | . 46046 | . 88768 | . 47588 | . 87951 | . 49116 | . 87107 | 35 |
| 26 |  | . 42946 | . 90309 | . 44516 | . 89545 | . 46072 | . 88755 | . 47614 | . 87937 | . 49141 | . 87093 | 34 |
| 27 |  | . 42972 | . 90236 | . 415 | . 89533 | . 46097 | . 88741 | . 47639 | . 87923 | . 49166 | . 87079 | 33 |
| 9 |  | . 42999 | . 90234 | . 44568 | . 89519 | . 46123 | .88728 | . 47665 | . 87909 | . 49192 | . 87064 | 32 |
| 29 |  | . 43025 | .97271 | . 44594 | . 89506 | . 46149 | .88715 | . 47690 | . 87896 | . 49217 | . 87050 | 31 |
| 30 |  | . 43051 | . 90259 | . 44620 | . 89493 | . 46175 | . 88701 | . 47716 |  | . 4924 | . 87036 | 30 |
| 31 |  | . 43077 | . 90246 | . 44646 | . 89480 | . 46201 | . 88688 | . 47741 | . 87868 | . 49268 | . 87021 | 29 |
| 32 |  | . 43104 | . 90233 | . 41672 | . 89467 | .46923 | . 880674 | . 47767 | . 87854 | . 49293 | . 87007 | 28 |
| 33 |  | . 43130 | .90221 | . 44698 | . 89454 | . 46252 | . 88661 | . 47793 | . 87840 | . 49318 | . 86993 | 27 |
| 34 |  | . 43156 | . 90208 | . 44724 | . 89441 | .46278 | . 88647 | . 47818 | . 87826 | . 49344 | . 86978 | 26 |
| 35 |  | . 43182 | . 90196 | . 44750 | . 89128 | . 46304 | . 88634 | . 47844 | . 87812 | . 49369 | . 86964 | 25 |
| 36 |  | . 43209 | . 90183 | . 44776 | . 89415 | . 46330 | . 88620 | . 47869 | . 87798 | . 49394 | . 86949 | 24 |
| 37 |  | . 43235 | . 90171 | . 44802 | . 89402 | . 46355 | . 88607 | . 47895 | . 87784 | . 49419 | . 86935 | 23 |
| 38 |  | . 43221 | . 90158 | . 44828 | . 893389 | . 46381 | . 88593 | . 47920 | . 87777 | . 49445 | . 86921 | 21 |
| 39 |  | . 432 | . 90146 | . 44854 | . 89376 | . 40407 | . 88580 | . 47946 | . 87756 | . 49470 | . 86906 | 21 |
| 40 |  | . 4 | . 90133 |  | . 89363 | . 46433 | 6 | . 47971 | . 87 | 5 | . 86892 | 20 |
| 41 |  | . 43340 | . 90120 | . 44906 | . 89350 | . 46458 | . 88553 | . 47997 | . 87729 | . 49521 | . 86878 | 19 |
| 42 |  | . 43366 | . 90108 |  | . 893337 | . 46484 | . 88539 | . 48022 | . 87715 | . 49546 | . 86883 | 18 |
| 43 |  | . 43392 | . 90095 | . 44958 | . 89334 | . 46510 | . 885276 | . 48048 | . 87701 | . 49571 | . 86849 | 17 |
| 44 |  | . 43418 | . 900082 | . 44984 | . 89311 | . 46536 | . 88512 | . 48073 | . 87687 | . 49596 | . 868834 | 16 |
| 45 |  | . 43445 | . 90070 | . 45010 | .89298 | . 46561 | . 88499 | . 48099 | .87673 | . 49622 | . 86820 | 15 |
| 46 |  | . 43471 | . 90057 | . 45036 | . 89285 | . 46587 | . 88485 | . 48124 | . 87659 | . $49 \times 47$ | . 86805 | 14 |
| 47 |  | . 43497 | . 90045 | . 45062 | .89272 | . 46613 | . 884772 | . 48150 | . 87645 | . 49672 | . 86791 | 13 |
| 48 |  | . 43523 | . 90032 | . 45088 | . 89259 | . 46639 | . 88458 | . 48175 | . 87631 | . 49697 | .86777 | 12 |
| 49 |  | . 43549 | . 90019 | . 45114 | . 89245 | . 46664 | . 88445 | . 48201 | . 87617 | . 49723 | . 86762 | 11 |
| 50 |  | . 435 | . 900 | . 45140 | .89232 | . 46690 | . 88431 | . 48226 | . 87603 | $.49748$ | . 86748 | 10 |
| 51 |  | . 43602 | . 89994 | . 45166 | . 89219 | . 46716 | . 88417 | . 48252 | . 87589 | . 49773 | . 86733 | 9 |
| 2 |  | . 43628 | . 89981 | . 15192 | . 89206 | . 40742 | . 88404 | . 48227 | . 87575 | . 49798 | . 86719 | 8 |
|  |  | . 43654 | . 89968 | . 45218 | . 89193 | . 46767 | . 88390 | . 48303 | . 87561 | . 49824 | . 86704 | 7 |
|  |  | . 43680 | . 89956 | . 45243 | . 89180 | . 46793 | . 883377 | . 48328 | . 87546 | . 49849 | . 86690 | 6 |
| 55 |  | . 43706 | . 89943 | . 45269 | . 89167 | . 46819 | . 88363 | . 48354 | . 87532 | . 49874 | . 86675 | 5 |
|  |  | . 43733 | . 89930 | . 45295 | . 89153 | . 46844 | . 88349 | . 48379 | . 87518 | . 49899 | . 86661 | 4 |
|  |  | . 43759 | . 89918 | . 45321 | . 89140 | . 46870 | . 883336 | . 48405 | . 87504 | . 49924 | 86646 | 3 |
|  |  | . 43785 | . 89905 | . 45347 | . 89127 | . 46896 | . 88322 | . 48430 | . 87490 | . 49950 | 86632 | 2 |
| 50 |  | . 43811 | . 89892 | 453\%3 | . 89114 | . 46921 | . 888308 | . 48456 | 87476 | . 49975 | 86617 | 1 |
|  |  | . 4 | . 8 | . 45399 | . 89 | . 46947 | . 8 | . 48481 | . 8 |  | . 86603 | 0 |
|  |  | Co | ine | Cosi | Sine | si | Sin |  | Sin | osi | SI |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

TABLE VI.-Continued.
Natural Sines and Cosines.


TABLE VI.-Continued.
Natural Sines and Cosines.

|  | $35^{\circ}$ |  | $36^{\circ}$ |  |  |  | $38^{\circ}$ |  | $39^{\circ}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Sine | Cosin | Sine |  | Sine |  | Sine | Cosin | Sine |  |  |
| 0 | . 573588 | - 81 | . 58 |  | . 60 | . 798 | 61566 | .78801 | 62932 | T | 60 |
| 1 | . 57381 | . 81899 |  |  | . 60205 | . 798946 | . 61589 | . 78783 | . 622955 | . 77696 | 59 |
| 2 | . 57405 | . 81888 | . 588826 |  | . 60228 | .79829 | 61612 | . 788765 | . 632977 | .776\%8 | 58 |
| 3 |  |  |  |  |  |  |  |  |  |  | 7 |
| 4 | . 57453 | . 81848 | . 588 | . 808 | . 60274 | . 79793 | . 61658 | . 78729 | . 63022 | . 77641 | 56 |
| 5 | . 57477 | . 81832 | . 58896 | . 80816 | . 60298 | 79776 | . 61681 | 78711 | . 63045 | . 77623 |  |
| 6 | . 57501 | . 81815 | . 58920 | . 80799 | . 60321 | . 7975 | . 61704 | 78694 | . 63068 | . 77605 |  |
| 7 | . 57524 | . 81798 | . 58943 | .80782 | . 60344 | . 79741 | . 61726 | 786\%6 | . 63000 | . 77586 | 3 |
| 8 | . 57548 | . 81782 |  | . 80765 | . 60367 | .79723 | . 61749 | . 78658 | . 63113 |  |  |
| 10 | . 5775 | . 81765 | . 58990 | . 807 | . 60 | 7970 | . 61772 | . 78640 | . 63135 | . 77550 | 51 |
| 10 | . 5 |  |  |  |  |  |  |  |  |  | 50 |
| 11 | . 5761 | . 817 | . 590 | . 80 | . 60 | 79671 | . 61818 | . 78604 | . 63180 | . 77513 | 40 |
| 12 | . 57643 | . 8171 | . 59061 | . 80696 | . 60460 | 79653 | . 61841 | . 78586 | . 63203 | . 77494 |  |
| 13 | . 57667 | . 81698 | . 59084 | . 80679 | . 60483 | 79635 | . 61864 | . 78568 | . 63225 | . 77476 | 4 |
| 14 | . 57691 | . 81681 | . 59108 | . 806 | . 60506 | \%96 | . 61887 | . 785 | . 63248 | .77458 | 46 |
| 15 | . 57715 | . 81664 | . 59131 | . 80644 | . 60529 | 79600 | . 61909 | . 78533 | . 63271 | . 77439 | 45 |
| 16 | . 57738 | . 81647 | . 59154 | . 80627 | . 60553 | 79583 | . 61932 | . 78514 | . 63293 | . 77421 | 44 |
| 17 | . 57762 | . 81631 | . 59178 | . 80610 | . 60576 | 79565 | . 61955 | 78196 | . 63316 | . 77402 | 43 |
| 18 | . 57786 | . 81614 | . 59201 | . 80593 | . 60599 | . 7954 | . 61978 | .78478 | . 6333 | . 77384 | 42 |
| 19 | . 57810 | . 81597 | .5922 | .805\%6 | . 60622 | . 79530 | .6200 | 78460 | . 63361 |  |  |
| 20 | . 57 |  |  | . 80 |  | . 79 |  |  |  | . 77347 | 40 |
| 21 | . 5785 | . 81563 | . 59272 | . 80541 | . 60668 | 79494 | . 62046 | . 78424 | . 63406 | . 77329 | 39 |
| 22 | . 57881 | . 8154 | . 5929 | . 80524 | . 60691 | .794\% | . 62069 | . 78405 | . 63428 | . 77310 |  |
| 23 | . 57904 | . 81530 | . 59318 | . 80507 | . 60714 | 79459 | . 62092 | . 7838 | . 63451 | . 77292 |  |
| 24 | . 57928 | . 81513 | . 59342 | . 80489 | . 60738 | . 9441 | . 62115 | . 78369 | . 63473 | . 77273 | 6 |
| 25 | . 57959 | . 81496 | . 59365 | .80472 | . 60761 | 79424 | . 62138 | . 78351 | . 6349 | . 772 | 35 |
| 23 | . 579 | . 81479 | . 59389 | . 804 | . 60784 | . 79406 | . 62160 | .7833 | . 63518 | . 77236 |  |
| 27 | . 579 | . 81462 | . 59412 | . 80438 | . 60807 | 79388 | . 62183 | . 78315 | . 63540 | . 77218 | 33 |
| 28 | . 58 | . 81445 | . 59436 | . 804 | . 608 | . 7937 | . 62206 | . 78297 | . 63563 | . 77199 |  |
| 29 | . 5804 | . 81428 | . 59459 | . 80403 | 60853 | . 79353 | . 62229 | . 78279 | . 6358 | . 77181 | 1 |
| 30 | . 58 | . 81412 | . 59482 | . 803 | . 60876 | 79335 | . 62251 | r8261 | . 636 | . 77162 | 30 |
| 31 | . 58094 | . 81395 | . 59 | . 803 | . 60 | 79318 | . 62274 | . 78243 | . 63630 | . 77144 | 29 |
| 3 | . 58118 | . 81378 | . 59529 | . 80351 | . 60922 | 79300 | . 62297 | . 78225 | . 63653 | .77125 | 28 |
| 33 | . 58141 | . 81361 | . 59552 | . 80334 | . 60945 | . 9282 | . 62320 | . 78206 | . 63675 | . 77107 | 27 |
| 34 | . 58 | . 81344 | . 59576 | . 80316 | . 609 | . 79264 | . 62342 | . 78188 | . 6369 |  | 6 |
| 35 | . 58189 | . 81327 | . 59599 | . 80299 | . 60991 | . 79247 | . 62365 | . 78170 | . 63720 | . 77070 | 25 |
| 36 | . 58212 |  | .59622 | .80282 | . 61015 | 79229 | . 62388 | . 78152 | . 63742 | . 77051 | 24 |
| 37 | . 582 | . 81293 | . 59646 | . 80264 |  | 79211 | . 62411 | . 78134 | 6376 | . 77033 | 23 |
| 38 | . 58 | . 81276 | . 59669 | . 80247 | . 61061 | 79193 | . 62433 | . 78116 | 6378 | . 77014 | 22 |
| 39 | . 58883 | .81259 | . 59693 | . 80230 | . 6 | . 79176 | . 62456 | . 78098 | . 63810 | . 76996 | 21 |
| 40 | . 58307 | . 81242 |  |  |  |  |  |  |  | . 76 | 20 |
| 41 | . 5833 | .81225 |  |  |  |  |  |  | . 63854 | 76959 | 19 |
| 42 | . 58354 | . 81208 | . 59763 | . 80178 | . 61153 | . 79122 | . 62524 | .78043 | . 63877 |  | 18 |
| 43 | . 58378 | . 81191 | . 597 | . 80160 | . 61176 | . 79105 | . 62547 | . 78025 | . 63899 | . 76921 | 17 |
| 44 | . 58401 | . 81174 | . 59809 | . 80143 | . 61199 | .79C87 | .625\%0 | . 78007 | . 63922 | . 76903 | 16 |
| 45 | . 58125 | . 81157 |  | . 80125 | . 61222 | 79069 | . 62592 | . 77988 | . 63944 | - | 15 |
| 46 | . 58449 | . 81140 | . 59856 | . 80108 | . 61245 | 79051 | . 62615 | . 77970 | . 63966 | . 76866 | 14 |
| 47 | . 58472 | . 81123 | . 59879 | . 80091 | . 61268 | . 79033 | . 62638 | . 77952 | . 63989 | . 76847 | 13 |
| 48 | . 58496 | . 81106 |  | . $800 \%$ | . 61291 | . 79016 | . 62660 | . 77934 | 64611 |  | 11 |
| 49 | . 58519 | . 81089 | . 59926 | . 80056 | . 61314 | . 78998 | . 62683 | . 77916 | 64033 | 76810 | 11 |
| 50 |  |  |  | . 80038 |  |  |  |  |  | . $76 \%$ | 10 |
| 51 |  |  |  | . 8002 | . 61360 | . 78962 | . 62728 | . 77879 | . $640 \div 8$ | . 767 |  |
| 52 | . 58590 | . 81038 | . 59995 | . 80003 | . 61383 | . 78944 | . 62751 | . 77861 | . 64100 | . $76 \%$ |  |
| 53 | . 58614 | . 81021 | . 60019 | . 79986 | . 61406 | . 78926 | . 62774 | . 77843 | . 64123 | . 76735 |  |
| 54 | . 58 | . 81004 | . 60042 | . 79968 | . 61429 | . 78908 | . 62796 | . 77824 | . 64145 | .76\%17 |  |
| 55 | . 58661 | . 80987 | . 60065 | . 99951 | . 61451 | . 78891 | . 62819 | . 77806 | . 64167 | 7669 |  |
| 56 | . 58888 | . 80970 | . 60089 | . 79934 | . 61474 | . 78873 | . 62842 | . 77788 | . 64190 | . 6667 |  |
| 5 | . 587708 | . 80953 | . 60112 | . 79916 | . 61497 | . 7885 | . 62864 | . 77769 | . 64212 | .6661 |  |
| 58 | . 58731 | . 80936 | . 60135 | .79899 | . 61520 | . 78837 | . 6288 | . 77751 | . 64234 | . 76642 |  |
| 59 | . 587755 | . 8091 | . 60158 | . 798 | . 61543 | . 78 | . 62909 | . 7773 | - | 76623 |  |
| 60 |  | - |  | . 79864 |  | T | 6293 | . 77715 | . 64279 | 76604 | 0 |
|  |  | Sine |  | Sine |  |  |  |  | Cos | Sine |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

## TABLE VI.-Continued.

Natural Sines and Cosines.


## TABLE VII.

natural Tangents and Cotangents.

|  | $0^{\circ}$ |  | $1{ }^{\circ}$ |  | $2{ }^{\circ}$ |  | $3{ }^{\circ}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang |  |
|  | . 00000 | Infinite. | . 01746 | 57.2900 | . 03492 | 28.6363 | . 05241 | 19.0811 | $\overline{60}$ |
|  | . 00029 | 3437.75 | . 01775 | 56.3506 | . 03551 | 28.3994 | . 05270 | 18.9755 | 59 |
|  | . 00058 | 1718.87 | . 01804 | 55.4415 | . 03550 | 28.1664 | . 05299 | 18.8711 | 58 |
|  | . 00087 | 1145.92 | . 01833 | 54.5613 | . 03579 | 27.9372 | . 05328 | 18.7678 | 57 |
|  | . 00116 | 859.436 | . 01862 | 53.7086 | . 03609 | 27.7117 | . 05357 | 18.6656 | 56 |
|  | . 00145 | 687.549 | . 01891 | 52.8821 | . 03638 | 27.4899 | . 05387 | 18.5645 | 55 |
| 6 | . 00175 | 572.957 | . 01920 | 52.0807 | . 036667 | ${ }_{27}^{27.2715}$ | . 05416 | 18.4645 | 54 |
|  | .00204 .00233 | 491.106 429.718 | . 01949 | 51.3032 50.5485 | . 03696 | 27.0566 26.8450 | . 05445 | 18.3655 18.2677 | 53 |
|  | . 00262 | 381.971 | . 02007 | 49.8157 | .03\%54 | 26.6367 | . 05503 | 18.1708 | 51 |
| 10 | . 00291 | 343.774 | . 02036 | 49.1039 | . 03783 | 26.4316 | . 05533 | 18.0750 | 50 |
| 1 | . 00320 | 312.521 | . 02066 | 48.4121 | . 03812 | 26.2296 | . 05562 | 17.9802 | 49 |
| 2 | . 00349 | 286.478 | . 02095 | 47.7395 | . 03842 | 26.0307 | . 05591 | 17.8863 | 48 |
| 3 | . 00378 | 264.441 | . 02124 | 47.0853 | . 03871 | 25.8348 | . 05620 | 17.7934 | 47 |
| 4 | . 00407 | 245.552 | . 02153 | 46.4489 | . 03900 | 25.6418 | . 05649 | 17.7015 | 46 |
| 15 | . 00433 | 229.182 | . 02182 | 45.8294 | . 03929 | 25.4517 | . 05678 | 17.6106 | 45 |
| 6 | . 00465 | 214.858 | . 02211 | 45.2261 | . 03958 | 25.2644 | . 05708 | 17.5205 | 44 |
| 17 | . 00495 | 202.219 | . 02240 | 44.6386 | . 03987 | 25.0798 | .05737 | 17.4314 | 43 |
|  | . 00524 | 190.984 | . 02269 | 44.0661 | . 04016 | 24.8978 | . 05766 | 17.3432 | 42 |
| 9 | . 00553 | 180.932 | . 02298 | 43.5081 | . 04046 | 24.7185 | .05795 | 17.2558 | 41 |
| 20 | . 00582 | 171.885 | . 02328 | 42.9641 | . 04075 | 24.5418 | . 05824 | 17.1693 | 40 |
| 2122232425262627282930 | . 00611 | 163.700 | . 02357 | 42.4335 | . 04104 | 24.3675 | . 05854 | 17.0837 | 39 |
|  | . 00640 | 156.259 | . 02386 | 41.9158 | . 04133 | 24.1957 | . 05883 | 16.9990 |  |
|  | . 00669 | 149.465 | .02415 | 41.4106 | . 04162 | 24.0263 | . 05912 | 16.9150 | 37 |
|  | . 00698 | 143.237 | . 02144 | $40.91 \% 4$ | . 04191 | 23.8593 | . 05941 | 16.8319 | 36 |
|  | . 00727 | 137.507 | . 02473 | 40.4358 | . 04220 | 23.6945 | . 05970 | 16.7496 | 35 |
|  | . 00756 | 132.219 | .02502 | 39.9655 | . 04250 | 23.5321 | . 05999 | 16.6681 | 34 |
|  | . 00785 | 127.321 | . 02531 | 39.5059 | . 04279 | 23.3718 | . 06029 | 16.5874 | 33 |
|  | . 00815 | 122.754 | . 02560 | 39.0568 | . 04308 | 23.2137 | . 06058 | 16.5075 | 32 |
|  | . 00844 | 118.540 | . 02589 | 38.6177 | . 01337 | 23.0577 | . 06087 | 16.4283 | 31 |
|  | . 00873 | 114.589 | . 02619 | 38.1885 | . 0436 | 22.9038 | . 06116 | 16.3499 | 30 |
| 31 | . 00902 | 110.892 | . 02648 | 37.7686 | . 04395 | 22.7519 | . 06145 | 16.272 | 29 |
| 32 | . 00931 | 107.426 | . 02677 | 37.3579 | . 04424 | 22.6020 | . 06175 | 16.1952 | 28 |
|  | . 00960 | 104.171 | . 03706 | 36.9560 | . 04454 | 22.4541 | . 06204 | 16.1190 | 27 |
|  | . 00989 | 101.107 | .02\%35 | 36.5627 | . 04483 | 22.3081 | . 06233 | 16.0435 | 26 |
| 34 35 | . 01018 | 98.2179 | .02\%64 | 36.176 | . 04512 | 22.1640 | . 06262 | 15.9687 | 25 |
| ${ }_{37}^{36}$ | . 01047 | 95.4895 | .02793 | 35.8006 | . 04541 | 22.0217 | . 06291 | 15.8945 | 24 |
|  | . 01076 | 92.9085 | . 02822 | 35.4313 | . 04570 | 21.8813 | . 06321 | 15.8211 | 23 |
|  | . 01105 | 90.4633 | . 02851 | 35.0695 | . 04599 | 21.7426 | . 06350 | 15.7483 | 22 |
| $\left\|\begin{array}{l} 38 \\ 39 \\ 40 \end{array}\right\|$ | . 01135 | 88.1436 | . 02881 | 34.7151 | . 04628 | 21.6056 | . 06379 | 15.6762 | 21 |
|  | . 01164 | 85.9398 | . 02910 | 34.36\%8 | . 04658 | 21.4704 | . 06408 | 15.6048 | 20 |
| 41 | . 01193 | 83.8435 | . 02939 | 34.0273 | . 04687 | 21.3369 | . 06437 | 15.5340 | 19 |
| 4 | . 01222 | 81.8470 | . 02963 | 33.6935 | . 04716 | 21.2049 | . 06467 | 15.4638 | 18 |
| 43 | . 01251 | 79.9434 | .02997 | 33.3662 | . 04745 | 21.0747 | . 06496 | 15.3943 | 17 |
| 44 | . 01280 | 78.1263 | . 03026 | 33.0452 | . 04774 | 20.9460 | . 06535 | 15.3254 | 16 |
| 45 | . 01309 | 76.3900 | . 03055 | 32.7303 | . 04803 | 20.8188 | . 06554 | 15.2571 | 15 |
|  | .C1338 | 74.7292 | . 03084 | 32.4213 | . 04833 | 20.6932 | . 06584 | 15.1893 | 14 |
| 7 | . 01367 | 73.1390 | . 03114 | 32.1181 | . 04862 | 20.5691 | . 06613 | 15.1222 | 13 |
| 48 | . 01396 | 71.6151 | . 03143 | 31.8205 | . 04891 | 20.4465 | . 06642 | 15.0557 | 12 |
|  | . 01425 | 70.1533 | . 03172 | 31.5284 | . 04920 | 20.3253 | . 06671 | 14.9898 | 11 |
| 50 | . 01455 | 68.7501 | . 03201 | 31.2416 | . 04949 | 20.2056 | . 06700 | 14.9244 | 10 |
| 5 5 | . 01484 | 67.4019 | . 03230 | 30.9599 | . 04978 | 20.0872 | .06730 | 14.8596 |  |
|  | . 01513 | 66.1055 | . 03259 | 30.6833 | . 05007 | 19.9702 | . 06759 | 14.7954 | 8 |
|  | . 01542 | 64.8580 | . 03288 | 30.4116 | . 05037 | 19.8546 | . 06788 | 14.7317 | 7 |
|  | . 01571 | 63.6567 | . 03317 | 30.1446 | . 05066 | 19.7403 | . 06817 | 14.6685 | 6 |
|  | . 01600 | 62.4992 | . 03346 | 29.8823 | . 05095 | 19.6273 | . 06847 | 14.6059 | 5 |
|  | . 01629 | 61.3829 | . 03376 | 29.6245 | . 05124 | 19.5156 | . 06876 | 14.5438 | 4 |
|  | . 01658 | 60.3058 | . 03405 | 29.3711 | . 05153 | 19.4051 | . 06905 | 14.4823 | 8 |
|  | . 01687 | 59.2659 | . 03434 | 29.1220 | . 05182 | 19.2959 | . 06934 | 14.4212 | 2 |
|  | . 01716 | 58.2612 | . 03463 | 28.8771 | . 05212 | 19.1879 | . 06963 | 14.3607 | , |
|  | . 01746 | 57.2900 | . 03492 | 28.6363 | . 05241 | 19.0811 | . 06993 | 14.3007 | 0 |
|  | Cotang Tang |  | $\overline{\text { Cotang Tang }}$ |  | Cobang | Tang | otan | Tang |  |
|  | $89^{\circ}$ |  | $88^{\circ}$ |  | $87^{\circ}$ |  | $86^{\circ}$ |  |  |

## TABLE VII.-Continued.

Natural Tangents and Cotangents.

|  | $4{ }^{\circ}$ |  | $5^{\circ}$ |  | $6^{\circ}$ |  | 70 |  | , |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang |  |
| 0 | . 06993 | 14.3007 | . 08749 | 11.4301 | . 10510 | 9.51436 | . 12278 | 8.14435 | $\overline{60}$ |
| 1 | .07022 | 14.2411 | . 08778 | 11.3919 | . 10540 | 9.48781 | . 12308 | 8.12481 | 59 |
| 2 | . 07051 | 14.1821 | . 08807 | 11.3540 | . 10569 | 9.46141 | . 12338 | 8.10536 | 58 |
| 3 | . 07080 | 14.1235 | . 08837 | 11.3163 | . 10599 | 9.43515 | . 12367 | 8.08600 | 57 |
| 4 | . 07110 | 14.0655 | . 08866 | 11.2789 | . 10628 | 9.40904 | .12397 | 8.06674 | 56 |
|  | . 07139 | 14.0079 | . 08895 | 11.2417 | . 10657 | 9.38307 | . 12426 | 8.04756 | 55 |
| 6 | . 07168 | 13.9507 | . 08925 | 11.2048 | . 10687 | 9.35724 | . 12456 | 8.02848 | 54 |
| 7 | . 07197 | 13.8940 | . 08954 | 11.1681 | . 10716 | 9.33155 | . 12485 | 8.00948 | 53 |
| 8 | . 07227 | 13.8378 | . 08983 | 11.1316 | . 10746 | 9.30599 | . 12515 | 7.99058 | 52 |
| 9 | . 07256 | 13.7821 | . 09013 | 11.0954 | . 10775 | 9.28058 | . 12544 | 7.97176 | 51 |
| 10 | . 07285 | 13.7267 | . 09042 | 11.0594 | . 10805 | 9.25530 | . 12574 | 7.95302 | 50 |
| 11 | . 07314 | 13.6719 | . 09071 | 11.0237 | . 10834 | 9.23016 | . 12608 | 7.93438 | 49 |
| 12 | . 07344 | 13.6174 | . 09101 | 10.9882 | . 10863 | 9.20516 | . 12633 | 7.91582 | 48 |
| 13 | . 07373 | 13.5634 | . 09130 | 10.9529 | . 10893 | 9.18038 | . 12662 | 7.89734 | 47 |
| 14 | . 07402 | 13.5098 | . 09159 | 10.9178 | . 10922 | 9.15554 | . 12692 | 7.87895 | 46 |
| 15 | . 07431 | 13.4566 | . 09189 | 10.8829 | . 10952 | 9.13093 | .12722 | 7.86064 | 45 |
| 16 | . 07461 | 13.4039 | . 09218 | 10.8483 | . 10981 | 9.10646 | . 12751 | 7.84242 | 44 |
| 17 | . 07490 | 13.3515 | . 09247 | 10.8139 | . 11011 | 9.08211 | . 12781 | 7.82428 | 43 |
| 18 | . 07519 | 13.2996 | . 09277 | 10.7797 | . 11040 | 9.05789 | . 12810 | 7.80622 | 42 |
| 19 | . 07548 | 13.2480 | . 09306 | 10.7457 | .11070 | 9.03379 | . 12840 | 7.78825 | 41 |
| 20 | . 07578 | 13.1969 | . 09 | 10.7119 | . 11099 | 9.00983 | . 12869 | 7.77035 | 40 |
| 21 | . 07607 | 13.1461 | . 09365 | 10.6783 | . 1112 | 8.98598 | . 12899 | 7.75254 | 39 |
| 22 | . 07636 | 13.0958 | . 09394 | 10.6450 | . 11158 | 8.96227 | . 12929 | 7.73480 | 38 |
| 23 | . 07665 | 13.0458 | . 09423 | 10.6118 | . 11187 | 8.93867 | . 12958 | 7.71715 | 37 |
| 24 | . 07695 | 12.9962 | . 09453 | 10.5789 | . 11217 | 8.91520 | . 12988 | 7.69957 | 36 |
| 25 | . 07724 | 12.9469 | . 09482 | 10.5462 | . 11246 | 8.89185 | . 13017 | 7.68208 | 35 |
| 26 | . 07753 | 12.8981 | . 09511 | 10.5136 | .11276 | 8.86862 | . 13047 | 7.66466 | 34 |
| 27 | . 07782 | 12.8496 | . 09541 | 10.4813 | . 11305 | 8.84551 | . 13076 | 7.64732 | 33 |
| 28 | . 07812 | 12.8014 | . 09570 | 10.4491 | . 11335 | 8.82252 | .13106 | 7.63005 | 32 |
| 29 | . 07841 | 12.7536 | . 09600 | 10.4172 | . 11364 | 8.79964 | . 13136 | 7.61287 | 31 |
| 30 | . 07870 | 12.7062 | . 09629 | 10.3854 | 11394 | 8.77689 | 43165 | 7.59575 | 30 |
| 31 | . 07899 | 12.6591 | . 09658 | 10.3538 | . 11423 | 8.75425 | . 13195 | 7.57872 | 29 |
| 32 | . 07929 | 12.6124 | . 09688 | 10.3224 | . 11452 | 8.73172 | . 13224 | 7.56176 | 28 |
| 33 | . 07958 | 12.5660 | . 09717 | 10.2913 | . 11482 | 8.70931 | . 13254 | 7.54487 | 27 |
| 34 | . 07987 | 12.5199 | . 09746 | 10.2602 | . 11511 | 8.68701 | . 13284 | 7.57806 | 26 |
| 35 | . 08017 | 12.4742 | . 09776 | 10.2294 | . 11541 | 8.66482 | . 13313 | 7.51132 | 25 |
| 36 | . 08046 | 12.4288 | . 09805 | 10.1988 | . 11570 | 8.642\% 5 | . 13343 | 7.49465 | 24 |
| 37 | . 08075 | 12.3838 | . 09834 | 10.1683 | . 11600 | 8.62078 | . 13372 | 7.47806 | 23 |
| 38 | . 08104 | 12.3390 | . 09864 | 10.1381 | . 11629 | 8.59893 | . 13402 | 7.46154 | 22 |
| 39 | . 08134 | 12.2946 | . 09893 | 10.1080 | . 11659 | 8.57718 | . 13432 | 7.44509 | 21 |
| 40 | . 08163 | 12.2505 | . 09923 | 10.0780 | . 11688 | 8.55555 | . 13461 | 7.42871 | 20 |
| 41 | . 08192 | 12.2067 | . 0 | 10.0483 | . 11718 | \& 53402 | . 13491 | 7.41240 | 19 |
| 42 | . 08221 | 12.1632 | . 09981 | 10.0187 | . 11747 | 8.51259 | . 13521 | 7.39616 | 18 |
| 43 | .08251 | 12.1201 | . 10011 | 9.98931 | . 11777 | 8.49128 | . 13550 | 7.37999 | 17 |
| 44 | . 08280 | 12.0\%72 | . 10040 | 9.96007 | . 11806 | $8.4 \% 007$ | . 13580 | 7.36389 | 16 |
| 45 | . 08309 | 12.0346 | . 10069 | 9.93101 | . 11836 | 8.44896 | . 13609 | 7.34786 | 15 |
| 46 | . 08339 | 11.9923 | . 10099 | 9.90211 | . 11865 | 8.42795 | . 13639 | 7.33190 | 14 |
| 47 | . 08368 | 11.9504 | . 10128 | 9.87338 | . 11895 | 8.40705 | . 13669 | 7.81600 | 13 |
| 48 | . 08397 | 11.9087 | . 10158 | 9.84482 | . 11924 | 8.88625 | . 13698 | 7.30018 | 12 |
| 49 | . 08427 | $11.86 \%$ | . 10187 | 9.81641 | . 11954 | 8.36555 | . 13778 | 7.28442 | 11 |
| 50 | . 08456 | 11.8262 | . 10216 | 9.78817 | . 11983 | 8.34496 | . 13758 | 7.26873 | 10 |
| 51 | . 08485 | 11.7853 | . 10246 | 8.76009 | . 12013 | 8.32446 | . 13787 | 7.25310 | 9 |
| 52 | . 08514 | 11.7448 | . 10275 | 9.73217 | . 12042 | 8.30406 | . 13817 | 7.233754 | 8 |
| 53 | . 08544 | 11.7045 | . 10305 | 9.70441 | . 12072 | 8.28376 | . 13846 | 7.22204 | 7 |
| 54 | . 08573 | 11.6645 | . 10334 | 9.67680 | . 12101 | 8.26355 | . 13876 | 7.20661 | 6 |
| 55 | . 08602 | 11.6248 | . 10363 | 9.64935 | . 12131 | 8.24345 | . 13906 | 7.19125 | 4 |
| 56 | . 08632 | 11.5853 | . 10393 | 9.62205 | . 12160 | 8.22344 | . 13935 | 7.17594 | 4 |
| 58 | . 08661 | 11.5461 | . 10422 | 9.59490 | . 12190 | 8.20352 | . 13965 | 7.16071 | 3 |
| 58 | . 08690 | 11.5072 | . 10452 | 9.56791 | . 12219 | 8.18370 | . 13995 | 7.14553 7.13042 | 1 |
| 59 | . 08720 | 11.4685 | . 10481 | 9.54106 | . 12249 | 8.16398 | . 14024 | 7.13042 7.11537 | 1 |
| 60 | . 08749 | 11.4301 | 10510 | 9.51436 | . 12278 | 8.14435 | . 14054 | 7.11537 | 0 |
|  | Cotang | Tang | Cotang Tang |  | $\overline{\text { Cotang }}$ | Tang | Cotang | Tang |  |
|  | $85^{\circ}$ |  | $84^{\circ}$ |  | $83^{\circ}$ |  | $82^{\circ}$ |  |  |

TABLE VII.-Continued.
Natural Tangents and Cotangents.

|  | $8^{\circ}$ |  | $9{ }^{\circ}$ |  | $10^{\circ}$ |  | $11^{\circ}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang |  |
| 0 | 0 .14054 | 7.11537 | . 15838 | 6.31375 | . 17633 | 5.67128 | . 19438 | 5.14455 | $\overline{60}$ |
|  | 1.14084 | 7.10038 | . 15868 | 6.30189 | . 17663 | 5.66165 | . 19468 | 5.13658 | 59 |
| 2 | . 14113 | 7.08546 | . 15898 | 6.29007 | . 17693 | 5.65205 | . 19498 | 5.12862 | 58 |
| 3 | 3.14143 | 7.07059 | . 15928 | 6.27829 | . 17723 | 5.64248 | . 19529 | 5.12069 | 57 |
|  | . 11173 | 7.05579 | . 15958 | 6.26655 | . 17753 | 5.63295 | . 19559 | 5.11279 | 56 |
| 5 | . 14202 | 7.04105 | . 15988 | 6.25486 | . 17783 | 5.62344 | . 19589 | 5.10490 | 55 |
| 6 | 6.14232 | 7.02637 | . 16017 | 6.24321 | . 17813 | 5.61397 | . 19619 | 5.09704 | 54 |
| 7 | . 14262 | 6.91174 | . 16047 | 6.23160 | . 178843 | 5.60452 | . 19649 | 5.08921 | ${ }_{59}^{53}$ |
| 8 | . 142931 | 6.99718 6.98268 | .16077 .16107 | 6.22003 6.20851 | .17873 <br> .17903 | 5.59511 5.58573 | . 19680 | 5.08139 5.07360 | 52 |
| 10 | . 14351 | 6.96823 | . 16137 | 6.19703 | . 17933 | 5.57638 | . 19740 | 5.06584 | 50 |
| 11 | . 14381 | 6.95385 | . 16167 | 6.18559 | . 17963 | 5.56706 | . 19770 | 5.05809 | 49 |
| 12 | . 14410 | 6.93952 | . 16196 | 6.17419 | . 17993 | 5.55777 | . 19801 | 5.05037 | 48 |
| 13 | . 14440 | 6.92525 | . 16226 | 6.16283 | . 18023 | 5.54851 | . 19831 | 5.04267 | 47 |
| 14 | . 14470 | 6.91104 | . 16256 | 6.15151 | . 18053 | 5.53927 | . 19861 | 5.03499 | 46 |
| 15 | . 14499 | 6.89688 | . 16286 | 6.14023 | . 18083 | 5.53007 | . 19891 | 5.02734 | 45 |
| 16 | . 14529 | 6.88278 | . 16316 | 6.12899 | . 18113 | 5.52090 | . 19921 | 5.01971 | 44 |
| 17 | . 14559 | 6.86874 | . 16346 | 6.11779 | . 18143 | 5.51176 | . 19952 | 5.01210 | 43 |
| 18 | . 14588 | 6.85475 | . 16376 | 6.10664 | . 18173 | 5.50264 | . 19982 | 5.00451 | 42 |
| 19 | . 14618 | 6.81082 | . 16405 | 6.09552 | . 18203 | 5.49356 | . 20012 | 4.99695 | 41 |
| 20 | . 14648 | 6.82694 | . 16435 | 6.08444 | . 18233 | 5.48451 | . 20042 | 4.98940 | 40 |
| 21 | . 14678 | 6.81312 | . 16465 | 6.07340 | . 18263 | 5.47548 | . 20073 | 4.98188 | 39 |
| 22 | . 14707 | 6.79936 | . 16495 | 6.06240 | . 18293 | 5.40048 | . 20103 | 4.97438 |  |
| 23 | . 14737 | 6.78564 | . 16525 | 6.05143 | . 18323 | 5.45751 | . 20133 | 4.96690 | 37 |
| 24 | . 14767 | 6.77199 | . 16555 | 6.04051 | . 18353 | 5.44857 | . 20164 | 4.95945 | 36 |
| 25 | . 14796 | 6.75838 | . 16585 | 6.02962 | . 18384 | 5.43966 | . 20194 | 4.95201 | 35 |
| 26 | . 14826 | 6.74483 | . 16615 | 6.01878 | . 18414 | 5.43077 | . 20224 | 4.94460 | 34 |
| 27 | . 14856 | 6.73133 | . 16645 | 6.00797 | . 18444 | 5.42192 | . 20254 | 4.93721 | 33 |
| 28 | . 14886 | 6.71789 | . 16674 | 5.99720 | . 18474 | 5.41309 | . 20235 | 4.92984 | 32 |
| 2 | . 14915 | 6.70450 | . 16704 | 5.93646 | . 18504 | 5.40429 | . 20315 | 4.92249 | 1 |
| 30 | . 14945 | 6.69116 | . 16734 | 5.97576 | . 18534 | 5.39552 | . 20345 | 4.91516 | 30 |
| 31 | . 14975 | 6.67787 | . 16764 | 5.96510 | . 18564 | 5.38677 | . 20376 | 4.90785 | 29 |
| 32 | . 15005 | 6.66463 | . 16794 | 5.95448 | . 18594 | 5.37805 | . 20406 | 4.90056 | 28 |
| 33 | . 15034 | 6.65144 | . 16824 | 5.94390 | . 18624 | 5.36936 | . 20436 | 4.89330 | 27 |
| 34 | . 15064 | 6.63831 | . 16854 | 5.93335 | . 18654 | 5.36070 | . 20466 | 4.88605 | 26 |
| 35 | . 15094 | 6.62523 | . 16884 | 5.92283 | . 18684 | 5.35206 | . 20497 | 4.87882 | 5 |
| 36 | . 15124 | 6.61219 | . 16914 | 5.91236 | . 18714 | 5.34345 | . 20527 | 4.87162 | 24 |
| 37 | . 15153 | 6.59921 | . 16944 | 5.90191 | . 18745 | 5.33487 | . 20557 | 4.86444 | 23 |
| 38 | . 15183 | 6.58627 | .16974 | 5.89151 | . 18775 | 5.32631 | . 20588 | 4.85727 | 2 |
| 39 | . 15213 | 6.57339 | . 17034 | 5.88114 | . 18805 | 5.31778 | . 20618 | 4.85013 | 21 |
| 40 | . 15243 | 6.56055 | . 17033 | 5.87080 | . 18835 | 5.30928 | . 20648 | -4.84300 | 20 |
| 41 | . 15272 | 6.54777 | 17063 | 5.86051 | . 18865 | 5.30080 | . 20679 | 4.83590 | 19 |
| 42 | . 15302 | 6.53503 | . 17093 | 5.85024 | . 18895 | 5.29235 | . 20709 | 4.82882 | 18 |
| 43 | . 15332 | 6.52234 | . 17123 | 5.84001 | . 18925 | 5.28393 | . 20739 | 4.82175 | 17 |
| 44 | . 15362 | 6.50970 | . 17153 | 5.82982 | . 18955 | 5.27553 | . 20770 | 4.81471 | 16 |
| 45 | . 15391 | 6.49710 | . 17183 | 5.81966 | . 18986 | 5.26715 | . 20800 | 4.80769 | 15 |
| 46 | . 15421 | 6.48456 | . 17213 | 5.80953 | . 19016 | 5.25880 | . 20830 | 4.80068 | - |
| 47 | . 15451 | 6.47206 | . 17243 | 5.79944 | . 19046 | 5.25048 | . 20861 | 4.79370 | 13 |
| 48 | . 15481 | 6.45961 | . 17273 | 5.78938 | . 19076 | 5.24218 | . 20891 | 4.78673 | 12 |
| 49 | . 15511 | 6.44720 | . 17303 | 5.77936 | . 19106 | 5.23391 | . 20921 | 4.77978 | 11 |
| 50 | . 15540 | 6.43484 | . 17 | 5.76937 | . 19136 | 5.22566 | . 209 | 4.77286 | 10 |
| 51 | . 15570 | 6.42253 | . 17363 | 5.75941 | . 19166 | 5.21744 | . 20982 | 4.76595 |  |
| 53 | . 15600 | 6.41026 | . 17393 | 5.74949 | . 19197 | 5.20925 | . 21013 | 4.75906 | 8 |
| 53 | . 15630 | ${ }^{6.39804}$ | . 17423 | 5.73960 | . 19227 | 5.20107 | . 21043 | 4.75219 | 7 |
| 54 | . 15660 | 6.38587 | . 17453 | 5.72974 | . 19257 | 5.19293 | . 21073 | 4.74534 | 6 |
| 55 | . 15689 | 6.37374 | . 17483 | 5.71992 | . 19287 | 5.18480 | . 21104 | 4.73851 | 5 |
| 56 | . 15719 | 6.36165 | . 17513 | 5.71013 | . 19317 | 5.17671 | . 21134 | 4.73170 | 4 |
| 57 | . 15749 | 6.34961 | . 17543 | 5.70037 | . 19347 | 5.16863 | . 21164 | 4.72490 | 8 |
| 58 | . 15779 | 6.33761 | . 17573 | 5.69064 | . 19378 | 5.16058 | . 21195 | 4.71813 | 2 |
| 5 | . 15809 | 6.32566 | 17603 | 5.68094 | . 19408 | 5.15256 | .21225 | 4.71137 | 1 |
| 60 | . 15838 | 6.31375 | . 17633 | 5.67128 | . 19438 | 5.14455 | . 21256 | 4.70463 | 0 |
|  | Cotang | Tang | Cotang | Tang | $\overline{\text { Cotang }}$ | Tang | Cotang | Tang |  |
|  |  |  |  |  |  |  |  |  |  |

TABLE VII.-Continued.
Natural Tangents and Cotangents.

|  | $12^{\circ}$ |  | $13^{\circ}$ |  | $14^{\circ}$ |  | $15^{\circ}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ang | Cotang | Ta | Cotang | Tang | Cotang | Tang | Cotang |  |
| 0 | . 212 | 4.70463 | . 2308 | 4.33148 | . 2493 | 4.01078 | . 26795 | 3.73205 | 0 |
|  | . 21286 | 4.69 | . 23117 | 4.32573 |  | 4.00582 |  | 3.72771 | 59 |
|  | . 21316 | 4.69121 | . 23148 | 4.32001 | . 2499 | 4.00086 | . 268 | 3.ヶ2338 | 58 |
| 3 | . 21347 | 4.68452 | . 23179 | 4.31430 | . 25026 | 3.99592 | . 26888 | 3.71907 | 57 |
| 4 | . 21377 | 4.67786 | . 23209 | 4.30860 | . 25056 | 3.99099 | . 26920 | 3.71476 | 56 |
|  | . 2140 | 4.67121 | . 23240 | 4.30291 | . 2508 | 3.98607 | . 26951 | 3.71046 |  |
|  | . 2143 | 4.6645 | . 23271 | 4.29\%74 | . 25118 | 3.98117 | . 2698 | 3.70616 |  |
|  | . 21469 | 4.65797 | . 23301 | 4.29159 | . 25149 | 3.97627 | . 27013 | 3.70188 | 53 |
|  | . 21499 | 4.6513 | 23 | 4.28595 | . 25180 | 3.97139 | . 270 | 3.69761 | 52 |
|  | . 21 | 4.64 | . 23363 | 4.28032 | . 25 | 8.9665 |  | 3.6833 |  |
|  | . 21 | 4.63 |  |  | . 25242 |  |  | 3. | 50 |
| 11 | . 215 | 4.631 | . 23 | 4.26911 | . 252 | 3.95680 | 27 | 3.68485 | 49 |
| 12 | . 21621 | 4.6251 |  | 4.26352 | . 2530 | 3.95196 | . 2710 | 3.68061 | 48 |
|  | . 21651 | 4.6186 | . 23 | 4.25795 | . 25335 | 3.91713 | . 27201 | 3.67688 | 47 |
| 14 | . 21682 | 4.61219 | . 23516 | 4.25239 | . 25366 | 3.94232 | . 27232 | 3.67217 | 46 |
| 15 | . 21712 | 4.60572 | . 23547 | 4.24685 | . 25397 | 3.93751 | . 2726 | 3.66796 | 45 |
| 16 | . 21743 | 4.599 |  | 4.24132 | . 2542 | 3.93271 | .2\%2 | 3.663r6 | 44 |
| 17 | . 21773 | 4.59283 | . 2360 | 4.23580 | . 25459 | 3.92793 | . 2732 | 3.6 | 43 |
| 18 | . 2180 | 4.58641 | . 2363 | 4.23030 | . 25490 | 3.93316 | . 2735 | 3.65538 | 42 |
| 19 | . 21834 | 4.5800 | . 2367 | 4.22481 | . 25521 | 3.91839 | . 27 | 3.65121 |  |
| 20 | . 21864 | 4.57363 | 2370 | 4.21933 | . 2555 | 3.9136 | . 27419 | 3.6 |  |
| 21 | . 2189 | 4.5 | . 2373 | 4.21387 | . 2558 | 3.90890 | 27 | 3.64289 | 39 |
| 22 | . 2192 | 4.56 | . 23 | 4.20842 | . 2561 | 3.90417 | . 27482 | 3.63874 | 38 |
| 23 | . 2195 | 4.55 | . 23 | 4.20298 | . 2564 | 3.89945 | . 27513 | 3.63461 |  |
|  | . 21986 | 4.548 | 288 | 4.19756 | . 256 | 3.8947 | . 275 | 3.6 |  |
| 25 | . 22017 | 4.54196 | . 2385 | 4.19215 | . 2570 | 3.89004 | . 2757 | 3.62636 |  |
| 26 | . 2204 | 4.5356 | . 2388 | 4.1867 | . 2573 | 3.88536 | . $2 \pi 60$ | 3.62224 |  |
|  | . 2207 | 4.6529 | 391 | 4.18137 | . 25769 | 3.88068 | . 2763 | 3.618 |  |
| 28 | . 22108 | 4.52316 | . 2394 | 4.17600 | . 25800 | 3.87601 | . 27670 | 3.61405 |  |
| 29 | . 21213 | 4.51693 | . 2397 | 4.17 | . 2588 | 3.871 | . 27701 | 3.60996 |  |
| 30 | . 22 | 4.51071 |  | 4.16530 | . 25862 |  | . 277 |  |  |
|  | . 22200 | 4. | 2403 | 4.15997 | . 25 | 3. | . 27 |  |  |
| 32 | . 2223 | 4.49832 | . 24069 | 4.1546 | . 25924 | 3.85745 | . $27 \%$ | 3.59775 |  |
| 33 | . 222 | 4.49215 | . 24100 | 4.14934 | . 2595 | 3.85284 | . 278 | 3.59370 |  |
| 34 | .22292 | 4.4860 | . 24131 | 4.14405 | . 25981 | 3.84824 | . 2788 |  |  |
| 35 | .22322 | 4.47986 | . 24162 | 4.13877 | . 26017 | 3.84364 | . 27889 | 3.58562 |  |
|  | . 223 | 4.4737 | . 2419 | 4.1335 | . 26048 | 3.839 | . 2792 | 3.58160 |  |
|  | . 2238 | 4.46 | . 2422 | 4.12825 | . 26019 | 3.83442 | . 279 |  |  |
|  | . 2241 | 4.4615 | . 2425 | 4.12301 | . 26110 | 3.82992 | . 2798 | 3.57357 | 22 |
|  | . 224 | 4.45 | . 24 | 4.1 | . 26141 | 3.8253 | . 280 | 3.5695\% | 21 |
| 40 | . 22 | 4.44942 |  |  | . 26172 | 3.82083 |  |  | 20 |
| 41 | . 2 |  |  |  |  |  |  |  | 18 |
|  | . 2253 | 4.4373 | 2437 | 4.10216 | . 26235 | 3.81177 | . 2810 | 3.55761 |  |
| 43 | . 2256 | 4.43134 | . 24408 | 4.09699 | . 2626 | 3.80726 | . 281 | 3.55364 | 17 |
| 44 |  | 4.4 | . 24439 | 4.09182 | . 2629 | 3.80276 | . 2817 | 3.5496 |  |
|  | . 2262 | 4.4 | . 24470 | 4.08666 | . 2632 | 3.7982 | . 282 | 3.545 |  |
| 46 | . 22658 | 4.41340 | . 24501 | 4.08152 | . 26359 | 3.79378 | . 2823 | 3.54179 | 14 |
|  | . 22689 | 4.40745 | . 2453 | 4.07639 | . 26390 | 3.78931 | 2820 |  |  |
|  | . 2271 | 4.40 | . 245 | 4.07127 | . 26421 | 3.78485 | . 28297 | 3.53393 |  |
|  | .22750 | 4.39560 | . 24593 | 4.06616 | . 2645 | 3.78040 | .28329 | 3.53001 | 11 |
| 50 | . 2278 | 4.38969 | . 2 | 4. | . 2648 | 3.77595 | . 28360 |  | 10 |
|  |  |  |  |  |  |  | 析 |  |  |
|  | .2281 | 4.3779 | 24681 | 4.05092 | . 2654 | 3.76709 | . 2842 | 3.51829 |  |
|  | . 22872 | 4.37207 | . 24717 | 4.04586 | . 26577 | 3.76268 | . 2845 | 3.51441 |  |
|  | . 2290 | 4.36623 | . 2474 | 4.04081 | . 2660 | 3.75828 | . 28486 | 3.51 |  |
|  | . 22934 | 4.36040 | . 24778 | 4.03578 | . 26639 | 3.75388 | . 28517 | 3.5060 |  |
|  | . 22964 | 4.35459 | . 24809 | 4.03076 | . 26670 | S.74950 | . 28549 | 3.50279 |  |
|  |  | 4.34879 | . 24840 | 4.02574 | . 2670 | 3.74512 | . 2858 | 3.4989 |  |
|  | . 2302 | 4.343 | . 2487 | 4.02074 | . 2673 | 3.74075 | . 2861 | 3.4950 |  |
| 60 | . 23056 | 4.3371 4.331 | . 24902 | 4.01576 $4.010 \% 8$ | .2576 | 3.73640 <br> 3.73205 | . 288643 | $\begin{aligned} & 3.491 \\ & 8.487 \end{aligned}$ | 0 |
|  | Cotang |  |  | Tang | Cotang | Tang | Cotang | Tang |  |
|  |  |  |  |  |  |  |  |  |  |

## TABLE VII.-Continued.

Natural Tangents and Cotangents.

|  | $16^{\circ}$ |  | $17^{\circ}$ |  | $18^{\circ}$ |  | $19^{\circ}$ |  | , |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang |  |
| 0 | . 28675 | 3.48741 | . 30573 | 3.27085 | . 32492 | 3.07768 | . 34433 | 2.90421 | $\overline{60}$ |
|  | . 28706 | 3.48359 | . 30605 | 3.26745 | . 32524 | 3.07464 | . 34465 | 2.90147 | 59 |
| 2 | . 28738 | 3.47977 | . 30637 | 3.26406 | . 32555 | 3.07160 | . 34498 | 2.89873 | 58 |
| 3 | . 28769 | 3.47596 | . 30669 | 3.26067 | . 32588 | 3.06857 | . 34530 | 2.89600 | 57 |
| 4 | . 288800 | 3.47216 3.46837 | . 30700 | 3.25729 3.25392 | . 32621 | 3.06554 | . 34563 | 2.89327 | 56 |
| 5 | . 28884 | ${ }_{3} .46458$ | . 30764 | 3.25055 | . 326 | 3.05950 | . 34628 |  | 55 |
|  | . 28895 | 3.46080 | . 30796 | 3.24719 | . 32717 | 3.05649 | . 34661 | 2.88511 | 53 |
| 8 | . 28927 | 3.45703 | . 30828 | 3.24383 | . 32749 | 3.05349 | . 34693 | 2.88240 | 52 |
|  | . 28958 | 3.45327 | . 30860 | 3.24049 | . 32782 | 3.05049 | . 34726 | 2.87970 | 51 |
| 10 | . 28990 | 3.44951 | . 30891 | 3.23714 | . 32814 | 3.04749 | . 34758 | 2.87700 | 50 |
| 11 | . 29021 | 3.44576 | . 30923 | 3.23381 | . 32846 | 3.04450 | . 34791 | 2.87430 | 49 |
| 12 | . 29053 | 3.44202 | . 30955 | 3.23048 | . 32878 | 3.04152 | . 34824 | 2.87131 | 48 |
| 13 | . 29084 | 3.43829 | . 30987 | 3.22715 | . 32911 | 3.03854 | . 34856 | 2.86892 | 47 |
| 14 | . 29116 | 3.43456 | . 31019 | 3.22384 | . 32943 | 3.03556 | . 34889 | 2.86624 | 46 |
| 15 | . 29147 | 3.43084 | . 31051 | 3.22053 | . 32975 | 3.03260 | . 34922 | 2.86356 | 45 |
| 16 | . 29179 | 3.42713 | . 31083 | $3.21 \% 22$ | . 33007 | 3.02963 | . 34954 | 2.86089 | 44 |
| $1 \%$ | . 29210 | 3.42343 | . 31115 | 3.21392 | . 33040 | 3.02667 | . 34987 | 2.85822 | 43 |
| 13 | . 29242 | 3.41973 | . 31147 | 3.21063 | . $330 \% 2$ | 3.02372 | . 35020 | 2.85555 | 42 |
| 19 | . 29224 | 3.41604 | . 31178 | 3.20734 | . 33104 | 3.02077 | . 35052 | 2.85289 | 41 |
| 20 | . 29305 | 3.41236 | . 31210 | 3.20406 | . 33136 | 3.01783 | . 35085 | 2.85023 | 40 |
| 21 | . 29337 | 3.40869 | . 31242 | 3.20079 | . 33169 | 3.01489 | . 35118 | 2.84758 | 39 |
| 2 | . 29368 | 3.40502 | . 31274 | 3.19752 | . 33201 | 3.01196 | . 35150 | 2.84494 | 38 |
| 23 | . 29400 | 3.40136 | . 31306 | 3.19426 | . 33233 | 2.00903 | . 35183 | 2.84229 | 37 |
| 24 | . 29132 | 3.39771 | . 31338 | 3.19100 | . $332 \mathrm{C6}$ | 3.00611 | . 35216 | 2.83965 | 36 |
| 25 | . 29463 | 3.39406 | . 31370 | 3.187\% | . 33298 | 3.00319 | . 35248 | 2.83702 | 35 |
| 26 | . 29495 | 3.39042 | . 31402 | 3.13451 | . 23330 | 3.00028 | . 35281 | 2.83439 | 34 |
| 27 | . 29526 | 3.38679 | . 31434 | 3.18127 | . 33363 | 2.99738 | . 35314 | 2.83176 | 33 |
| 28 | . 29558 | 3.38317 | . 31466 | 3.17804 | . 33395 | 2.99447 | . 35346 | 2.82914 | 32 |
| 29 | . 29590 | 3.37955 | . 31498 | 8.17481 | . 23427 | 2.99158 | . 35379 | 2.82653 | 31 |
| 30 | . 29621 | 3.37594 | . 3153 | 3.17159 | . 33460 | 2.98868 | . 354 | 2.82391 | 30 |
| 31 | . 29653 | 3.37234 | . 31562 | 3.16838 | . 38492 | 2.98580 | . 35445 | 2.82130 | 29 |
|  | . 29685 | 3.36875 | . 31594 | 3.10517 | . 33524 | 2.98292 | . 35477 | 2.81870 | 28 |
| 33 | . 29716 | 3.36516 | . 31626 | 3.16197 | . 33557 | 2.98004 | . 35510 | 2.81610 | 27 |
| 34 | . 29748 | 3.36158 | . 31658 | 3.15877 | . 33589 | 2.97717 | . 35543 | 2.81350 | 26 |
|  | . 29780 | 3.35800 | . 31690 | 3.15558 | . 33621 | 2.97430 | . 35576 | 2.81091 | 25 |
| 36 | . 29811 | 3.35443 | . 31722 | 3.15240 | . 33654 | 2.97144 | . 35608 | 2.80833 | 24 |
| 37 | . 29843 | 3.35087 | . 31754 | 3.14922 | . 33686 | 2.96858 | . 35641 | 2.80574 | 23 |
| 38 | . 29875 | 3.34732 | . 31786 | 3.14605 | . 33718 | $2.965 \% 3$ | . 35674 | 2.80316 | 22 |
| 39 | . 29906 | 3.34377 | . 31818 | 3.14288 | . 33751 | 2.96288 | . 35707 | 2.80059 | 21 |
| 40 | . 29938 | 3.34023 | . 31850 | 3.13972 | . 33783 | 2.96004 | . 35740 | 2.79802 | 20 |
| 41 | . 29970 | 3.33670 | . 31882 | 3.13656 | . 33816 | 2.95721 | . 35772 | 2.79545 | 19 |
|  | . 30001 | 3.33317 | . 31914 | 3.13341 | . 33848 | 2.95437 | . 35805 | 2.79289 | 18 |
| 43 | . 30033 | 3.32965 | . 31946 | 3.13027 | . 33881 | 2.95155 | . 35838 | 2.79033 | 17 |
| 44 | . 30065 | 3. 32614 | . 31978 | 3.12713 | . 33913 | 2.94872 | . 35871 | 2.78778 | 16 |
| 45 | . 30097 | 3.32264 | . 32010 | 3.12400 | . 33945 | 2.94591 | . 35904 | 2.78523 | 15 |
| 46 | . 30128 | 3.31914 | . 32042 | 3.12087 | . 33978 | 2.94309 | . 35937 | 2.78269 | 14 |
| 47 | . 30160 | 3.31565 | . 32074 | 3.11775 | . 34010 | 2.94028 | . 35969 | 2.78014 | 13 |
|  | . 30192 | 3.31216 | . 82106 | 3.11464 | . 34043 | 2.93748 | . 36002 | 2.77761 | 12 |
| 40 | . 30224 | 3.30868 | . 32139 | 3.11153 | . 34075 | 2.93468 | . 36035 | 2.77507 | 11 |
| 50 | . 30255 | 3.30521 | . $321 \% 1$ | 3.10842 | . 34108 | 2.93189 | . 36068 | 2.77254 | 10 |
| 51 | . 30287 | 3.30174 | . 32203 | 3.10532 | . 34140 | 2.92910 | . 36101 | 2.77002 | 9 |
|  | . 30319 | 3.29829 | . 32235 | 3.10223 | . 34173 | 2.92632 | . 36134 | 2.76750 | 8 |
| 53 | . 30351 | 3.29483 | . 32267 | 3.09914 | . 34205 | 2.92354 | . 36167 | 2.76498 | 7 |
| 54 | . 30382 | 3.29139 | . 32299 | 3.09606 | . 34238 | $2.920 \sim 6$ | . 36199 | 2.76247 | 6 |
|  | . 30414 | 3.28795 | . 32331 | 3.09298 | . 34270 | 2.91799 | . 36232 | 2.75996 | 5 |
| 5 | . 30446 | 3.28452 | . 32363 | 3.08991 | . 34303 | 2.91523 | . 36265 | $2.75 \% 46$ | 4 |
| 5 | . 30478 | 3.28109 | . 32396 | 3.08685 | . 34335 | 2.91246 | . 36298 | 2.75496 | 3 |
|  | . 30509 | 3.27767 | . 32428 | 3.08379 | . 34368 | 2.90971 | . 36331 | 2.75246 | 2 |
|  | 30541 | 3.27426 | . 32460 | 3.08073 | . 34400 | 2.90696 | . 36364 | 2.74997 | 1 |
| 60 | 30573 | 3.27085 | . 32492 | 3.07768 | . 34433 | 2.90421 | 36397 | 2.74748 | 0 |
|  | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang |  |
|  |  |  |  |  |  |  | 7 | $0^{\circ}$ |  |

TABLE VII. - Continued.
Natural Tangents and Cotangents.

|  | $20^{\circ}$ |  | $21^{\circ}$ |  | $22^{\circ}$ |  | $23^{\circ}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Tang | Cota | Tang |  | Ta | Cotang | Tang | Cotang |  |
|  | 0 ${ }^{\text {a }} 363897$ |  |  | 2.6 | . 40403 | 2.47509 | . 42447 | 2.35585 | $\overline{60}$ |
|  | ${ }_{2}^{1}{ }^{1} .3644$ | ${ }_{2}^{2.74499}$ | . 38453 | 2.60 | . 40436 | ${ }_{2}^{2.47302}$ | 2 | 2.35395 | 59 |
|  | 3 . 364 | 2.74004 | . 38487 | ${ }_{2.59831}^{2.6057}$ | . 40504 | 2.478888 | . 4251551 | 2.35205 | 5 |
|  | 4.36529 | 2.73756 | . 38 | 2.59606 | . 40538 |  |  | ${ }_{2} 2.34825$ | 56 |
|  | 5.38562 | 2.735 | . 385 |  | . 405 | 2.464 | . 42619 | 2.34636 | 55 |
|  |  |  | . 38 | 2.591 | . 40606 | 2.46270 | . 42654 | 2.34447 | 54 |
|  | 7 8.36 | ${ }^{2.73017}$ | . 38680 | 2.58932 | . 40640 | 2.46005 | . 42688 | 2.34 | 53 |
|  | ${ }^{8} .366694$ | 2.72771 | . 388687 | 2.5 | ${ }^{.40674} 4$ | 2.45860 |  |  |  |
|  | ${ }^{1}$. 36727 | ${ }_{2.72281}$ | . 38821 | 2.58261 | . 40741 | 2.4545 | . 42791 | 2.33693 | 50 |
|  | . 3676 | 2.720 | . 38 | 2.58 | . 40 | 2.45 | 428 | 2.33 | 49 |
|  |  |  |  | 2.57 | :4080 | 2.45043 | . 42860 | 2.33 | 48 |
|  | . 36 | ${ }_{2}^{2.7}$ | . 38 | ${ }^{2} .57593$ | . 40843 | 2.44839 | . 42894 | 2.33 | 7 |
|  |  |  |  |  |  |  |  |  |  |
|  |  | ${ }_{2}$ |  | 2. | . 40 | 2.44 | . 42 | 2.3 | 5 |
|  | . 3 | 2.\%0 | . 38 | 2.56 | . 40 | 2.4 | . 43032 | 2.3 | 3 |
|  |  |  | . 38988 | 2.56 | . 41013 | 2.43 | . 43067 | 2.3 | 2 |
|  |  | 2.70 |  | 2.5 | . 41047 | 2.43 | . 43101 | 2.3 | 41 |
|  | . 37057 | 2.698 | . 39055 | 2.56 | . 410 | 2.43 | . 43136 | 2.31 | 40 |
|  | . 37090 | 2.69612 | . 39089 | 2. | . 411 | 2.43 | 43170 | 2.31 | 39 |
|  | . 311 | 2.693 | . 39122 | 2.5 | . 411 | 2.43 | . 43 | 2. 31 | 8 |
|  | . 37 |  |  |  | . 411 | 2.42 |  |  |  |
|  | . 37190 | 2. | . 39 | 2.55 | . 41217 | 2.42 | . 43274 | 2.3 | 36 |
|  | . 3 | 2.68 | . 392 | ${ }_{2}^{2.54}$ | . 412285 | 2.42 | . 433343 |  | 55 |
|  | . 372 | 2.68175 | . 39290 | 2.545 | . 41319 | 2.42019 | . 43378 | 2.30534 | 33 |
|  | . 373 | ${ }^{2.6793}$ |  | 2.54 | . 41353 |  | . 43412 | 2.30 |  |
|  |  |  |  |  |  |  |  |  |  |
|  | 0 . 37388 | 2.67462 | . 3939 | 2.5 | . 414 | 2.41 | . 43481 | 2.29984 | 30 |
|  | . 37422 | 2.67225 | . 34425 | 2.53648 | . 41455 | 2.412 | 43516 | 2.29801 | 29 |
|  | . 374458 | 2.669 | . 33443 | ${ }^{2} .533$ | . 41490 | 2.410 | . 43550 | 2.29619 | ${ }^{8}$ |
|  |  |  |  |  | . 415 |  |  |  | 7 |
|  |  |  |  |  | . 415 |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  | . 375888 | 2.6 | ${ }^{3} 3959$ | ${ }^{2.52} 5$ | ${ }^{416660}$ | 2.4 | . 436 | 2.28891 | 24 |
|  |  | ${ }_{2.65}$ | . 393660 | 2.52142 | . 41694 | 2.39 | . 43758 | 2.28 | - |
|  |  | 2.65342 | . 39691 | 2.51929 | . 41728 | 2.396 | . 43793 | 2.28348 | 21 |
|  | . 37720 | 2.65109 | . 39727 | 2.51715 | . 41763 | 2.39449 | . 43828 | 2.28167 | 20 |
|  | . 37754 | 2.6 | . 39 | 2.51 | . 4179 | 2.39 | 43 | 2.27 | 19 |
|  | . 36 | 2.64642 |  | 2.5128 | . 4183 | 2.39 | . 438 | 2.278 | 18 |
|  | . 37 | 2.6 | . 39 | 2.5 | . 41865 | 2.38863 | ${ }^{.} 4339326$ | 2.27 | ${ }^{17}$ |
|  | . 37 | ${ }_{2.63945}$ | . 39 | ${ }_{2} .50652$ | ${ }^{41933}$ | ${ }_{2.38473}$ | ${ }^{.449001}$ | 2.22 | 15 |
|  | , | 2.63714 | . 399 | 2.504 | . 419 | 2.38 | . 440 | $2.2 \pi$ | 14 |
|  |  |  |  |  |  |  |  | 2.26 | 3 |
|  |  | 2.63252 | . 3993 | 2.50 | . 4203 |  | . 411 | 2.2 |  |
|  |  | ${ }_{2}^{2.630}$ | . 400 |  | . 42000 |  | . 4141 | 2.26 | 11 |
|  |  |  |  |  | . 42139 |  |  |  | 10 |
|  | . 38 | 2.62332 | . 40132 | 2.49177 | . 42173 | 2.37118 | . 442 | 2.26 | 8 |
|  | . 38153 | 2.62103 | . 40166 |  |  | 2.369 | . 442 | 2.25 | 7 |
|  |  | 2. | . 40 | 2.48 | 42242 | 2.36 | . 443 |  | 6 |
|  |  | 2.6 | ${ }^{40} 40234$ | 2.48549 | .42236 | 2 | . 44 |  | 5 |
|  |  | 2.61190 | . 40301 | 2.48132 | . 42345 | 2.361 | . 44418 | 2.25132 | 3 |
|  | ${ }^{38320}$ | 2.60963 | . 40335 | 2.47924 | . 42379 | 2.35967 | . 44453 | 2.24956 | 2 |
|  | ${ }^{38353}$ | 2.600 | 369 | 2.47716 | 42413 | 2.357 | 44488 | 2.24780 |  |
| 60 | . 38386 | 2.60509 | 40403 | 2.47509 | 42447 | 2.35585 | . 44523 | 2.24604 | 0 |
|  | Cot | ang | Cotang | an | Cotang | Tang | Cotang | Tan |  |
|  |  | $9^{\circ}$ |  | $8^{\circ}$ |  |  |  |  |  |

TABLE VII.-Continued.
Natural Tangents and Cotangents.

|  | 24* |  | $25^{\circ}$ |  | $26^{\circ}$ |  | $27^{\circ}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang |  |
| 0 | . 44523 | 2.24604 | . 46631 | 2.14451 | . 48773 | 2.05030 | . 50953 | 1.96261 | $\overline{60}$ |
|  | . 44558 | 2.24428 | . 46666 | 2.14288 | . 48809 | 2.04879 | . 50989 | 1.96120 | 59 |
| 2 | . 44593 | 2.24252 | . 46702 | 2.14125 | . 48845 | 2.04728 | . 51026 | 1.95979 | 58 |
| 3 | . 44627 | $2.240 \pi 7$ | . 46737 | 2.13963 | . 48881 | 2.04577 | . 51063 | 1.95838 | 57 |
| 4 | . 44662 | 2.23902 | . 46772 | 2.13801 | . 48917 | 2.04426 | . 51099 | 1.95698 | 56 |
| 5 | . 44697 | 2.23727 | . 46808 | 2.13639 | . 48353 | 2.04276 | . 51136 | 1.95557 | 55 |
| 6 | . 44732 | 2.23553 | . 46843 | 2.13477 | . 48989 | 2.04125 | . 51173 | 1.95417 | 54 |
| 7 | . 444807 | ${ }_{2}^{2.23378}$ | . 46879 | 2.13316 | . 49026 | 2.03975 | . 51209 | 1.95277 | ${ }^{53}$ |
| 9 | . 44837 | ${ }_{2} 2.23030$ | . 46959 | ${ }_{2}$ | . 490098 | ${ }_{2.036 \%}^{2.03825}$ | . 512483 | 1.95137 | 51 |
| 10 | . 44872 | 2.22857 | . 46985 | 2.12832 | . 49134 | 2.03526 | . 51319 | 1.94858 | 50 |
| 11 | . 44907 | 2.22683 | . 47021 | 2.12671 | . 49170 | 2.03376 | . 51356 | 1.94718 | 49 |
| 12 | . 44942 | 2.22510 | . 47056 | 2.12511 | . 49206 | 2.03227 | . 51393 | 1.94579 | 48 |
| 13 | . 44977 | 2.22337 | . 47092 | 2.12350 | . 49242 | 2.03078 | . 51430 | 1.94440 | 47 |
| 14 | . 45012 | 2.22164 | . 47128 | 2.12190 | . 49278 | 2.02929 | . 51467 | 1.94301 | 46 |
| 15 | . 45047 | 2.21992 | . 47163 | 2.12030 | . 49315 | 2.02780 | . 51503 | 1.94162 | 45 |
| 16 | . 45082 | 2.21819 | . 47199 | 2.11871 | . 49351 | 2.02631 | . 51540 | 1.94023 | 44 |
| 17 | . 45117 | 2.21647 | . 47234 | 2.11711 | . 49387 | 2.02483 | . 51577 | 1.93885 | 43 |
| 18 | . 45152 | 2.21475 | . 47270 | 2.11552 | . 49423 | 2.02335 | . 51614 | 1.93746 | 42 |
| 19 | . 45187 | 2.21304 | . 47305 | 2.11392 | . 49459 | 2.02187 | . 51651 | 1.93608 | 41 |
| 20 | . 45222 | 2.21132 | . 47341 | 2.11233 | . 49495 | 2.02039 | . 51688 | 1.93470 | 40 |
| 21 | . 45857 | 2.20961 | . 47377 | 2.11075 | . 49532 | 2.01891 | . 51724 | 1.93332 | 39 |
| 22 | . 45292 | 2.20790 | . 47412 | 2.10916 | . 49568 | 2.01743 | . 51761 | 1.93195 |  |
| 23 | . 45327 | 2.20619 | . 47448 | 2.10758 | . 49604 | 2.01596 | . 51798 | 1.93057 | 37 |
| 24 | . 45362 | 2.20449 | . 47483 | 2.10600 | . 49640 | 2.01449 | . 51835 | 1.92920 | 36 |
| 25 | . 45397 | 2.20278 | . 47519 | 2.10442 | . 49677 | 2.01302 | . 51872 | 1.92782 | 35 |
| 26 | . 45432 | 2.20108 | . 47555 | 2.10284 | . 49713 | 2.01155 | . 51909 | 1.92645 | 34 |
| 27 | . 45467 | 2.19938 | . 47590 | 2.10126 | . 49749 | 2.01008 | . 51946 | 1.92508 | 33 |
| 28 | . 45502 | 2.19769 | . 47626 | 2.09969 | . 49786 | 2.00862 | . 51983 | 1.92371 | 32 |
| 29 | . 45538 | 2.19599 | . 47662 | 2.09811 | . 49822 | 2.00715 | . 52020 | 1.92235 | 31 |
| 30 | . 455 \%3 | 2.19430 | . 47698 | 2.09654 | . 49858 | 2.00569 | . 52057 | 1.92098 | 30 |
| 31 | . 45608 | 2.19261 | . 47733 | 2.09498 | . 49894 | 2.00423 | . 52094 | 1.91962 | 29 |
| 32 | . 45643 | 2.19092 | . 47769 | 2.09341 | . 49931 | 2.00277 | . 52131 | 1.91826 | 28 |
| 33 | . 45678 | 2.18923 | . 47805 | 2.09184 | . 49967 | 2.00131 | . 52168 | 1.91690 | 27 |
| 34 | . 45713 | 2.1875 | . 47840 | 2.09028 | . 50004 | 1.99986 | . 52205 | 1.91554 | 26 |
| 35 | . 45748 | 2.18587 | . 47876 | $2.088 \%$ | . 50040 | 1.99841 | . 52242 | 1.91418 | 25 |
| 36 | . 45784 | 2.18419 | . 47912 | 2.08716 | . 50076 | 1.99695 | . 52279 | 1.91282 | ${ }_{2}^{24}$ |
| 37 | . 45819 | 2.18251 | . 47948 | 2.08560 | . 50113 | 1.99550 | . 52316 | 1.91147 | 23 |
| 38 | . 45854 | 2.13084 | . 47984 | 2.08405 | . 50149 | 1.99406 | . 52353 | 1.91012 | 22 |
| 39 | . 45889 | 2.17916 | . 48019 | 2.08250 | . 50185 | 1.99261 | . 52390 | 1.90876 | 21 |
| 40 | . 45924 | 2.17749 | . 48055 | 2.08094 | . 50222 | 1.99116 | . 52427 | 1.90741 | 20 |
| 41 | . 45960 | 2.17582 | . 48091 | 2.07939 | . 50258 | 1.98972 | . 52464 | 1.90607 | 19 |
| 42 | . 45995 | 2.17416 | . 48127 | 2.07785 | . 50295 | 1.98828 | . 52501 | 1.90472 | 18 |
| 43 | . 46030 | 2.17249 | . 48163 | 2.07630 | . 50331 | 1.98684 | . 52538 | 1.90337 | 17 |
| 44 | . 46065 | 2.17083 | . 48198 | 2.07476 | . 50368 | 1.98540 | . 52575 | 1.90203 | 16 |
| 45 | . 46101 | 2.16917 | . 48234 | 2.07321 | . 50404 | 1.98396 | . 52613 | 1.90069 | 15 |
| 46 | . 46136 | 2.16751 | . 48270 | 2.07167 | . 50441 | 1.98253 | . 52650 | 1.89935 | 14 |
| 47 | . 46171 | 2.16585 | . 48306 | 2.07014 | . 50477 | 1.98110 | . 52687 | 1.85801 | 13 |
| 48 | . 46206 | 2.16420 | . 48342 | 2.06860 | . 50514 | 1.97966 | . 52724 | 1.89667 | 12 |
| 49 | . 46242 | 2.16255 | . 48378 | 2.06706 | . 50550 | 1.97823 | . 52761 | 1.89533 | 11 |
| 50 | .462\%7 | 2.16090 | . 48414 | 2.06553 | . 50587 | 1.97681 | . 52798 | 1.89400 | 10 |
| 51 | . 46312 | 2.15025 | . 48450 | 2.06400 | . 50623 | 1.97538 | . 52836 | 1.89266 |  |
| 52 | . 46348 | 2.15760 | . 48186 | 2.06247 | . 50660 | 1.97395 | . 528873 | 1.89133 | 8 |
| 53 | . 46383 | 2.15596 | . 48521 | 2.06094 | . 50696 | 1.97253 | . 52910 | 1.89000 | 7 |
| 55 | . 46418 | 2.15432 | . 48557 | 2.05942 | . $50 ; 33$ | 1.97111 | . 52947 | 1.88867 | 6 |
| 55 | . 46454 | 2.15268 | . 48593 | 2.05790 | . 50769 | 1.96969 | . 52985 | 1.88734 | 5 |
| 56 | . 46489 | 2.15104 | . 48629 | 2.05637 | . 50808 | 1.96827 | . 53022 | 1.88602 | 4 |
| 5 | . $465 \% 5$ | 2.14940 | . 48665 | 2.05485 | . 50843 | 1.96685 | . 53059 | 1.88469 | 3 |
| 58 | . 46560 | $2.14 \% 7$ | . 48701 | 2.05333 | . 50879 | 1.96544 | . 53096 | 1.88337 | 2 |
| 59 | . 46595 | 2.14614 | . 48737 | 2.05182 | . 50916 | 1.96402 | . 53134 | 1.88205 | 1 |
| 60 | . 46631 | 2.14451 | . 48773 | 2.05030 | . 50953 | 1.96261 | 53171 | 1.88073 | 0 |
|  | Cotang | Tang | $\overline{\text { Cotang }}$ | Tang | Cotang | Tang | $\overline{\text { Cotang }}$ | Tang |  |
|  |  |  |  |  |  |  |  |  |  |

TABLE VII.-Continued.
Natural Tangents and Cotangents.

|  | $28^{\circ}$ |  | $29^{\circ}$ |  | $20^{\circ}$ |  | $31^{\circ}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Tang | Cotang | Tang | Cot | Tang | Co | Tang | g |  |
| 0 | . 5317 | 1.880 | . $55431{ }^{-1}$ | 1.80405 | . 57735 | 1.73 | . 60086 | 1.66428 | $\overline{60}$ |
|  | . 5322 | 1.87941 | . 55469 | 1.80281 | . 577874 | 1.73089 | . 60126 | 1.66318 | 58 |
|  | . 532 | ${ }_{1}^{1.88677}$ | . 55545 | 1.80034 | . 57851 | ${ }_{1} .72857$ | . 600205 | 1.66209 1.66099 | 58 |
|  | . 53 | 1.85046 | . 55583 | 1. 99911 |  | 1. 72 | . 60245 | 1.65 | 56 |
|  | 523 | 1.87415 |  | 1.7978 | .579 |  |  |  | 55 |
| 7 | .53393 | 1.87283 | . 5565697 | 1.19665 | .589607 | 1. 12509 | . 60 | ${ }_{1}^{1.65762}$ | ${ }_{5}^{54}$ |
|  | . 534450 | 1.87021 | . 55736 | 1.79419 | . 58046 | 1. | . 60403 | 1.65554 | 5 |
|  | . 53507 | 1.86891 | . 5574 | 1. 192926 | . 58085 | 1. 212163 | . 60443 | 1.65445 | 51 |
| 10 | . 53545 | 1.86760 | . 55812 | 1. 191 ז14 | . 58124 | 1.22047 | . 60483 | 1.65337 | 50 |
| 11 | . 533582 | 1.86 | . 55 | 1. 79051 | . 58 | 1.71932 | .60522 | 1.65228 | 49 |
|  | . 533627 | 1.86 |  | 1. 18929 | . 588210 | 1.7817 |  | ${ }_{1}^{1.65120}$ | 48 |
|  | . 53694 | 1.86239 | . 55954 | 1.78685 | . 588279 | 1.71588 | . 606042 | 1.654903 | 4 |
|  | .53732 | 1.86109 | . 56003 | 1.78563 | . 58 | 1.71473 | . 606 | 1.64795 | 45 |
|  |  | 1.85 | . 560 | 1. 28441 |  | 1.713 |  | 1.64687 | 4 |
|  | . 5 | 1.85 |  | 1.78 |  | 1.71244 |  | 1.64579 | 倍 |
|  | ${ }_{538}$ | 1.85 | ${ }^{.56}$ | 1. 1.8198 |  |  | . 608 |  | ${ }_{41}^{42}$ |
| 20 | . 53920 | 1.85462 | . 56194 | 1.77955 | . 58513 | 1. 50901 | . 60881 | 1.64256 | 40 |
| 21 | .53957 | 1.85333 | . 56232 | 1.7 | . 58 | 1.70787 | . 60921 | 1.64148 | 39 |
|  |  |  |  | 1.7 |  |  | . 609 |  | 38 |
|  |  |  |  | 1. 77 | . 58 | 1.70 | . 610 | 1.63934 | 37 |
|  | ${ }^{54070}$ | 1.84946 | ${ }^{.563}$ | 1.7471 |  | 1. 10446 |  |  | 36 |
|  | . 54145 | 1.846 | . 56424 | 1.77230 | . 587 | 1.:0219 | . 61120 | 1.63612 | 34 |
|  | . 54183 | 1.84561 | . 56462 | 1.77110 |  | 1.701 | . 61160 | 1.63505 | 33 |
|  |  |  |  |  |  | 1.69 |  |  |  |
|  |  |  |  | 768 |  | 1.6 |  |  |  |
| 30 | . 54296 | 177 | .56 | 1.767 | . 58 | 1.69 | . 61 | 1.63185 | 30 |
|  | .54333 | 1.84049 | . 56616 | 1.76629 | . 58944 | 1.69633 | . 61320 | 1.63079 | 29 |
|  | . 54371 | 1.83 | . 56654 | 1. 766510 |  | 1.69541 | . 61360 | 1.62972 |  |
|  | . 544 | 1.837 | . 56731 | 1. 1.762711 | . 590 | ${ }_{1}^{1.69428} 1$ | . 6141440 |  |  |
|  |  | 1.83 |  | 1.7615 | . 591 | 1.692 |  | 1.62654 | 5 |
|  |  |  |  | 1.760 | . 59140 | 1.690 | ${ }^{6} 6$ |  | 24 |
|  | . 545650 | 1.833 | . 5688 | 1. 75913 | . 59 | 1.68 | . 61561 | ${ }_{1}^{1.62442}$ | 3 |
|  | . 54635 | 1.83 | . 56 | 1. 1.7675 | . 592 | 1.68754 | . 61641 | 330 |  |
| 40 | . 54673 | 1.82906 | . 56962 | 1.75556 | . 59297 | 1.68643 | . 61681 | 1.62125 | 2 |
|  |  | 1.82 | . 578000 | 1.754 | . 593 | 1.685 | . 61 | 1.62 | 19 |
|  | . 54748 | 1.826 | . 5703 | 1.75319 | . 593 | 1.68419 | . 611 | 1.61 |  |
|  | . 54 | 1. | ${ }^{5}$ | 1.75 | . 59 | 1. | . 618181 | 1.618 | ${ }_{16}^{17}$ |
| 45 |  | 1.82276 | . 57155 | 1. 74964 | . 59494 | 1.68085 | . 61882 | 1.61 |  |
|  |  | 1.82150 | . 57193 | 1.74846 | . 5953 | 1.67974 | . 619 | 1.61493 | 4 |
|  |  |  | . 572 | 1.747 | 59 | 1.67 | . 61 |  | 3 |
|  |  |  | . 57271 | 1.74 | . 596 |  |  | 1.61 | 1 |
| 50 | . 55051 | 1.817649 1.818 | . 57348 | 1.74 | . 59 | 1. | 62 | 1.61074 | 10 |
| 51 | . 55089 | 1.81524 | . 57386 | 1.742 | . 59730 | 1.674 | . 621 | 1.60970 | 9 |
|  | . 55127 | 1.81399 | . 57425 | 1.741 | .59770 | 1.673 | . 6216 | 1.60865 |  |
|  | 5 | 1.812 | . 57464 | 1.7403 | . 598 | 1.671 | . 622 | 1.60 |  |
|  | . 5 | 1.81 | ${ }^{5} 575031$ | 1.739 | . 598 | 1.67 | . 62 | 1.60 | 6 |
|  |  | 1. |  |  |  |  | .623 | 1.60553 | 5 |
| 57 | . 55317 | 1.80777 | . 57619 | 1.73555 | . 599967 | 1.66757 | . 623 | 1.60345 | 3 |
|  | . 55355 | 1.80653 | . 57657 | 1.73438 | 60007 | 1.66647 | . 62406 | 1.60241 | 2 |
|  | 393 | 1.80529 | 696 | 1.73321 | 60046 | 1.66 | . 62446 | 1.60137 | 1 |
| 60 | 55431 | 1.80405 | . 57735 | 1.73205 | . 60086 | 1.66488 | . 62487 | 1.60033 | 0 |
|  | Co | Tang | Cotang | Tang | Co | Tan | tang | Tang |  |
|  |  | $1^{\circ}$ |  | $0^{\circ}$ |  |  |  |  |  |

TABLE VII.-Continued.
Natural Tangents and Cotangents.

|  | $32^{\circ}$ |  | $33^{\circ}$ |  | $34^{\circ}$ |  | $35^{\circ}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang |  |
| 0 | . 62487 | 1.60033 | . 64941 | 1.53986 | . 67451 | 1.48256 | . 70021 | 1.42815 | 60 |
| 0 | . 62527 | 1.59930 | . 64982 | 1.53888 | . 67493 | 1.48163 | . 70064 | 1.42726 | 59 |
| 玉 | . 62568 | 1.59826 | . 65084 | 1.53791 | . 67536 | 1.48070 | . 70107 | 1.42638 | 58 |
| 3 | . 62608 | 1.59723 | . 65065 | 1.53693 | . 67578 | 1.47977 | . 70151 | 1.42550 | 57 |
| 4 | . 62649 | 1.59620 | . 65106 | 1.53595 | . 67620 | 1.47885 | . 70194 | 1.42462 | 56 |
| 5 | . 62689 | 1.59517 | . 65148 | 1.53497 | . 67663 | 1.47792 | . 70238 | 1.42374 | 55 |
| 6 | .62730 | 1.59414 | . 65189 | 1.53400 | . 67705 | 1.47699 | . 70281 | 1.42286 | 54 |
| ${ }^{7}$ | . 62770 | 1.59311 | . 65231 | 1.53302 | . 67748 | 1.47607 | . 70325 | 1.42198 | 53 |
| 8 | . 62811 | 1.59208 | . 655272 | 1.53205 | . 67790 | 1.47514 | . 70368 | 1.42110 | 52 |
| 9 | . 62852 | 1.59105 | . 65314 | 1.53107 | . 67832 | 1.47422 | . 70412 | 1.42022 | 51 |
| 10 | . 62892 | 1.59002 | . 65355 | 1.53010 | . 67875 | 1.47330 | . 70455 | 1.41934 | 50 |
| 11 | . 62933 | 1.58900 | . 65397 | 1.52913 | . 67917 | 1.47238 | . 70499 | 1.41847 | 49 |
| 12 | . 62973 | 1.58797 | . 65438 | 1.52816 | . 6796 | 1.47146 | . 70542 | 1.41759 | 43 |
| 13 | . 63014 | 1.58695 | . 65480 | 1.52719 | . 68002 | 1.47053 | . 50586 | 1.41672 | 47 |
| 14 | . 63055 | 1.58593 | . 65521 | 1.52622 | . 68045 | 1.46902 | . 70629 | 1.41584 | 43 |
| 15 | . 63095 | 1.58490 | . 65563 | 1.52525 | . 6808 | 1.43870 | . 70673 | 1.41497 | 45 |
| 16 | . 63136 | 1.58388 | . 65604 | 1.52429 | . 68130 | 1.467\%8 | . 70717 | 1.41409 | 41 |
| 17 | . 63177 | 1.58886 | . 65646 | 1.52332 | . 68173 | 1.46686 | . 70760 | 1.41322 | 43 |
| 18 | . 63217 | 1.58184 | . 65688 | 1.52235 | . 68215 | 1.46595 | . 70804 | 1.41235 | 42 |
| 19 | . 63258 | 1.58083 | . 65729 | 1.52139 | . 68258 | 1.49503 | . 70348 | 1.41148 | 41 |
| 20 | . 632 | 1.57981 | . 65771 | 1.52043 | . 6830 | 1.46411 | . 708 | 1.41061 | 40 |
| 21 | . 63340 | 1.57879 | . 058 | 1.51946 | . 683 | 1.46320 | . 709 | 1.40974 | 9 |
| 22 | . 63380 | 1.57778 | . 65854 | 1.51850 | . 68336 | 1.46229 | . 70979 | 1.40887 | 83 |
| 23 | . 63421 | 1.57676 | . 65896 | 1.51 T04 | . 68429 | 1.46137 | . 71023 | 1.40300 | \% |
| 24 | . 63462 | $1.575 \%$ | . 65938 | 1.51658 | . 68471 | 1.46046 | . 71066 | 1.40714 | 36 |
| 25 | . 63503 | 1.57474 | . 65980 | 1.51502 | . 6351 | 1.45955 | . 71110 | $1.40<27$ | 5 |
| 26 | . 63544 | 1.57372 | . 66021 | 1.514C6 | . 68557 | 1.45864 | . 71154 | 1.40540 | 4 |
| 2 | . 63584 | 1.57271 | . 66063 | 1.51370 | . 68600 | 1.45773 | . 71198 | 1.40454 | 3 |
| 20 | . 63625 | 1.57170 | . 6610 | 1.51275 | . 6864 | 1.45682 | . 7124 | 1.40357 | 2 |
| 29 | . 63666 | 1.57069 | . 66147 | 1.51179 | . 68885 | 1.45592 | . 71285 | 1.40281 | 31 |
| 30 | . 63707 | 1.56969 | . 66183 | 1.51084 | . 6872 | 1.455 | . 713 | 1.40195 | 20 |
| 31 | . 63 | 1.5 | . 66230 |  |  | 1.4 | . 71 | 1.4 | 2 |
|  | . 6378 | 1.56767 | . $662{ }^{\circ}$ | 1.50893 |  | 1.453 | . 714 | 1.40 |  |
| 33 | . 63830 | 1.56667 | . 66314 | $1.50 \% 97$ | . 6885 | 1.45229 | . 71461 | 1.33936 | 27 |
| 34 | . 63871 | 1.56566 | . 66356 | 1.50702 | . 68900 | 1.45139 | . 71505 | 1.39850 | 26 |
|  | . 63912 | 1.56466 | . 66303 | 1.50607 | . 6894 | 1.45049 | . 71549 | 1.39764 | 25 |
| 36 | . 63953 | 1.56366 | . 66440 | 1.50512 | . 68985 | 1.44958 | . 71593 | 1.39679 | 24 |
| 37 | . 63994 | 1.56265 | .66482 | 1.50417 | . 69028 | 1.44868 | . 71637 | 1.39593 |  |
|  | . 64035 | 1.56165 | . 66521 | 1.50322 | . 69071 | 1.44778 | . 71681 | 1.39507 | 2 |
| 39 | . 64076 | 1.56065 | .66566 | 1.50228 | . 69114 | 1.44688 | . 71725 | 1.39421 | 21 |
| 40 | . 64 | 1.5 | . 66608 | 1.5 | . 69157 | 1. | . 71769 | 1.39336 | 20 |
| 41 | . 64158 | 1.558 | . 666 | 1.50038 | . 6920 | 1.44508 | . 71813 | 1.39250 | 19 |
| 4 | . 64199 | 1.55766 | . 66692 | 1.49944 | . 69243 | 1.44418 | . 71857 | 1.39165 | 13 |
| 43 | . 64240 | 1.55666 | . 66734 | 1.49849 | . 69288 | 1.44329 | . 71901 | 1.39079 | 17 |
| 44 | . 64281 | 1.55567 | . 66776 | 1.49755 | . 69339 | 1.44239 | . 71946 | 1.38994 | 16 |
| 45 | . 64322 | 1.55467 | . 66818 | 1.49661 | . 69311 | 1.44149 | - 71990 | 1.38909 | 1 |
| 46 | . 64363 | 1.55368 | . 66860 | 1.49566 | . 69416 | 1.44060 | . 72034 | 1.38824 | 14 |
|  | . 64404 | 1.55269 | . 66902 | 1.49472 | . 69459 | 1.43970 | . 72078 | 1.38738 | 13 |
| 48 | . 64446 | $1.551 \% 0$ | . 6694 | 1.49378 | . 69502 | 1.43881 | . 72122 | 1.38653 | 12 |
| 49 | 6448 | 1.55071 | . 60908 | 1.49284 | . | 1.43792 | .72167 | 1.38568 | 11 |
| 50 | . 64528 | 1.54972 | . 67028 | 1.49190 | . 6958 | 1.43703 | . 72211 | 1.38484 | 10 |
|  | . 64569 | 1.54873 | . 67071 | 1.49097 | . 69631 | 1.43614 | . 72255 | 1.38399 | 9 |
|  | . 64610 | 1.54774 | . 67113 | 1.49003 | . 69675 | 1.43525 | . 72299 | 1.38314 | 8 |
|  | . 64652 | 1.54675 | . 67155 | 1.48909 | . 69718 | 1.43436 | . 72344 | 1.38229 | 7 |
|  | 64693 | 1.54576 | . 67197 | 1.48816 | . 69761 | 1.43347 | . 72388 | 1.38145 | 6 |
|  | . 64734 | 1.54478 | . 677239 | 1.48722 | . 69804 | 1.43258 | . 72432 | 1.38060 | 5 |
|  | . 647817 | 1.54379 | . 67728 | 1.48629 | . 69884 | 1.43169 | . 72477 | 1.37976 | 4 |
|  | . 648178 | 1.54281 1.54183 | ${ }^{.} 67324$ | 1.48536 1.48442 | . 699993 | 1.43080 1.42992 | . 725 | 1.37891 1.37807 | 3 <br> 2 |
|  | . 64899 | 1.54085 | . 67409 | 1.48349 | . 69977 | 1.42903 | . 72610 | 1.37722 | 1 |
|  | 941 | 1.53986 | , | 1.48256 | O21 | 1.42815 | 硅 | 1.37638 | 0 |
|  | Cotan | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang |  |
|  |  | $57^{\circ}$ |  | $56^{\circ}$ |  | $55^{\circ}$ |  | $54^{\circ}$ |  |

TABLE VII.-Continued.
Natural Tangents and Cotangents.

|  | $36^{\circ}$ |  | $37^{\circ}$ |  | $38^{\circ}$ |  | $39^{\circ}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang |  |
| 0 | . 72654 | 1.37638 | . 75355 | 1.32704 | . 78129 | 1.27994 | . 80978 | 1.23490 | $\overline{60}$ |
| 1 | . 72699 | 1.37554 | . 75401 | 1.32624 | . 78175 | 1.27917 | . 81027 | 1.23416 | 59 |
| 2 | . 72743 | 1.37470 | . 75447 | 1.32544 | . 78222 | 1.27841 | . 81075 | 1.23343 | 58 |
| 3 | . 72788 | 1.37386 | . 75492 | 1.32464 | . 78269 | 1.27764 | . 81123 | 1.232\%0 | $5{ }^{8}$ |
| 4 | . 72832 | 1.37302 | . 75538 | 1.32384 | . 78316 | 1.27688 | . 81171 | 1.23196 | 56 |
| 5 | . 72887 | 1.37218 | . 75584 | 1.32304 | . 78363 | 1.27611 | . 81220 | 1.23123 | 55 |
| 6 | . 72921 | 1.37134 | . 75629 | 1.32224 | . 78410 | 1.27535 | . 81268 | 1.23050 | 54 |
| 7 | . 72966 | 1.37050 | . 75675 | 1.32144 | . 78457 | 1.27458 | . 81316 | 1.22977 | 53 |
| 8 | .73010 <br> .73055 | 1.36967 | . 75721 | 1.32064 | . 78504 | 1.27382 | . 81364 | 1.22904 | 52 |
| 9 | . 730105 | 1.36883 | . 75767 | 1.31984 | . 78551 | 1.27306 | . 81413 | 1.22831 | 51 |
| 10 | . 73100 | 1.36800 | . 75812 | 1.31904 | . 78598 | 1.27230 | . 81461 | $1.22 \% 58$ | 50 |
| 11 | . 73144 | 1.36716 | .75858 | 1.31825 | . 78645 | 1.27153 | . 81510 | 1.22685 | 49 |
| 12 | . 73189 | 1.36633 | . 75904 | 1.31745 | . 78892 | 1.2 207 | . 81558 | 1.22612 | 48 |
| 13 | . 73234 | 1.36549 | . 75950 | 1.31666 | . 78739 | $1.2 \pi 001$ | . 81606 | 1.22539 | 47 |
| 14 | . 73278 | 1.36466 | . 75996 | 1.31586 | . 78786 | 1.26925 | . 81655 | 1.22467 | 46 |
| 15 | . 73333 | 1.36383 | . 76042 | 1.31507 | . 78834 | 1.26849 | .81703 | 1.22394 | 45 |
| 16 | . 73368 | 1.36300 | .'6088 | 1.31427 | . 78881 | 1.26774 | . 81752 | 1.22321 | 44 |
| 17 | . 73413 | 1.36217 | . 76134 | 1.31348 | . 78928 | 1.26698 | . 81800 | 1.22249 | 43 |
| 18 | . 73457 | 1.36134 | . 76180 | 1.31269 | . 78975 | 1.20622 | . 81849 | $1.221 \% 6$ | 42 |
| 19 | . 73502 | 1.3CJ51 | .'62~6 | 1.31190 | . 79022 | 1.26546 | . 81898 | 1.22104 | 41 |
| 20 | . 73547 | 1.35968 | . 76272 | 1.31110 | . 79070 | 1.26471 | . 81946 | 1.22031 | 40 |
| 21 | . 73592 | 1.35885 | .76318 | 1.31031 | . 79117 | 1.26395 | . 81995 | 1.21959 | 39 |
| 22 | . 73637 | 1.35802 | . 76304 | 1.30952 | . 79164 | 1.20319 | . $8 \pm 044$ | 1.21886 | 38 |
| 23 | . 73681 | 1.35719 | . 76410 | 1.30373 | . 79212 | 1.26244 | . 82092 | 1.21814 | 37 |
| 24 | . 73776 | 1.35637 | . 76456 | 1.30795 | . 79259 | 1.26169 | . 82141 | 1.21742 | 36 |
| 25 | . 73771 | 1.35554 | . 76502 | 1.20716 | . 79308 | 1.26093 | . 82190 | $1.216{ }^{\text {r }} 0$ | 35 |
| 26 | . 73816 | 1.35472 | . 76548 | 1.30637 | . 79354 | 1.26018 | . 82238 | 1.21598 | 34 |
| 2 | .73861 | 1.35389 | . 76594 | 1.30558 | . 79401 | 1.25943 | . 82237 | 1.21526 | 33 |
| 28 | . 73906 | 1.35307 | . 76640 | 1.30480 | . 79449 | 1.25867 | . 82336 | 1.21454 | 32 |
| 29 | . 73951 | 1.35224 | .76686 | 1.30401 | . 79496 | 1.25702 | . 82385 | 1.21382 | 31 |
| 30 | . 73996 | 1.35142 | . 76733 | 1.30323 | . 79544 | 1.25717 | . 82434 | 1.21310 | 30 |
| 31 | . 74041 | 1.35060 | . 76779 | 1.30244 | . 79591 | 1.25642 | . 82483 | 1.21238 | 29 |
| 32 | . 74086 | 1.34978 | .76825 | 1.30166 | . 79639 | 1.25567 | . 82531 | 1.21166 | 28 |
| 33 | . 74131 | 1.34896 | . 76881 | 1.30087 | . 79636 | 1.25492 | . 82580 | 1.21094 | 27 |
| 25 | . 74176 | 1.34814 | . 76918 | 1.30009 | . 79734 | 1.25417 | . 82629 | 1.21023 | 26 |
| 35 | . $\% 4221$ | 1.34732 | . 76964 | 1.29931 | . $79 \% 81$ | 1.25343 | . 820678 | 1.20951 | 25 |
| 36 | . 74267 | 1.34650 | . 77010 | 1.29853 | . 79889 | 1.25268 | . 82727 | 1.20879 | 24 |
|  | . 74312 | 1.34568 | . 77057 | 1.29775 | . 79877 | 1.25193 | .82776 | 1.20808 | 23 |
| 38 | . 74357 | 1.34487 | . 77103 | 1. 220696 | . 79924 | 1.25118 | .82825 | 1.20736 | 22 |
| 3 | . 74402 | 1.31405 | . 77149 | 1.29618 | . 79972 | 1.25044 | . 82874 | 1.20665 | 21 |
| 40 | . 74447 | 1.34323 | 7196 | 1.29541 | . 800 | 1.24 | . 820 | 1.20593 | 20 |
| 41 | . 74492 | 1.34242 | . 77242 | 1.29463 | . 80067 | 1.24895 | .829\%2 | 1.20522 | 19 |
| 42 | . 74538 | 1.34160 | . 77239 | 1.29385 | . 80115 | 1.24820 | . 80022 | 1.20451 | 18 |
| 43 | . 74583 | 1.34079 | . 77335 | 1.29307 | . 80163 | $1.24 \% 46$ | . 83071 | 1.20379 | 17 |
| 45 | . 74628 | 1.33998 | . 77382 | 1.29229 | . 80211 | 1.24672 | . 83120 | 1.20308 | 16 |
| 45 | . 74674 | 1.33916 | . 77428 | 1.29152 | . 80258 | 1.24597 | . 83169 | 1.20237 | 15 |
| 46 | . 74719 | 1.33835 | . 77475 | 1.290\%4 | . 80306 | 1.24523 | . 83218 | 1.20166 | 14 |
| 48 | . 74764 | 1.33754 | . 77521 | 1.28997 | . 80354 | 1.24449 | . 83268 | 1.20095 | 13 |
| 48 | . 74810 | 1.33673 | . 77568 | 1.28919 | . 80402 | 1.24375 | . 83317 | 1.20024 | 12 |
| 49 | . 74855 | 1.33592 | . 77615 | 1.28842 | . 80450 | 1.24301 | . 83366 | 1.19953 | 11 |
| 50 | . 74 | 1.33511 | .77661 | 1.28764 | . 8049 | 1.24227 | . 83415 | 1.19882 | 10 |
| 51 | . 74949 | 1.33430 | . 77708 | 1.28687 | . 80546 | 1.24153 | . 83465 | 1.19811 | 9 |
|  | . 74991 | 1.33349 | . 77754 | 1.28610 | . 80594 | 1.24079 | . 83514 | 1.19740 | 8 |
|  | . 75037 | 1.83268 | . 77801 | 1.28533 | . 80642 | 1.24005 | . 83564 | 1.19669 | 7 |
|  | . 75082 | 1.33187 | . 77848 | 1.28456 | . 80690 | 1.23931 | . 83613 | 1.19599 | 6 |
|  | . 75128 | 1.33107 | . 77895 | 1.28379 | . 80738 | 1.23858 | . 838682 | 1.19528 | 5 |
|  | . 75173 | 1.33026 | . 77941 | 1.28302 | . 80786 | 1.23784 | . 83712 | 1.19457 | 4 |
|  | . 75219 | 1.32946 | . 77988 | 1.28225 | . 80834 | $1.2371 C$ | . 83761 | 1.19387 | 3 |
|  | . 75264 | 1.32865 | . 78035 | 1.28148 | . 80882 | 1.23637 | . 83811 | 1.19316 | 2 |
|  | . 75310 | 1.32785 | . 78082 | 1.28071 | . 80930 | 1.23563 | . 83880 | 1.19246 | 1 |
| 60 | 75355 | 1.32704 | . 78129 | 1.27994 | . 80978 | 1.23490 | . 83910 | 1.19175 | 0 |
|  | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | , |
|  |  | $3^{\circ}$ |  | 兂 | 5 | $1{ }^{\circ}$ |  |  |  |

TABLE VII.-Continued.
Natural Tangents and Cotangents.

|  | $40^{\circ}$ |  | $41^{\circ}$ |  | $42^{\circ}$ |  | $43^{\circ}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang |  |
| $\overline{0}$ | . 83910 | 1.19175 | . 86929 | 1.15037 | . 90040 | 1.11061 | . 93252 | 1.07237 | $\overline{60}$ |
| 1 | . 83960 | 1.19105 | . 86980 | 1.14969 | . 90093 | 1.10996 | . 93306 | 1.07174 | 59 |
| 2 | . 84009 | 1.19035 | . 87031 | 1.14902 | . 90146 | 1.10931 | . 93360 | 1.07112 | 58 |
| 3 | . 84059 | 1.18964 | . 87082 | 1.14834 | . 90199 | 1.10867 | . 93415 | 1.07049 | 57 |
| 4 | . 81108 | 1.18894 | . 87133 | 1.14767 | . 90231 | 1.10802 | . 93469 | 1.06987 | 56 |
| 5 | . 81158 | 1.18824 | . 87184 | 1.14699 | . 90304 | 1.10737 | . 93554 | 1.06925 | 55 |
| 6 7 | . 8121208 | 1.18754 1.18684 | . 8782387 | 1.14632 1.14565 | .90357 .90410 | 1.10672 1.10607 | . 9357878 | 1.06862 1.06800 | 54 |
| 8 | . 84307 | 1.18614 | . 87338 | 1.14498 | . 90463 | 1.10543 | . 93688 | 1.06738 | 5 |
| 9 | . 84357 | 1.18544 | . 87389 | 1.14430 | . 90516 | 1.104\%8 | . 93142 | 1.06676 | 51 |
| 10 | . 84407 | $1.184 \% 4$ | . 87441 | 1.14363 | . 00569 | . 1.10414 | . 93797 | 1.06613 | 50 |
| 11 | . 84457 | 1.18404 | . 87492 | 1.14296 | . 90621 | 1.10349 | . 93852 | 1.06551 | 49 |
| 12 | . 84507 | 1.18334 | . 87543 | 1.14229 | . $906 \% 4$ | 1.10285 | . 93906 | 1.06489 | 43 |
| 13 | . 84556 | 1.18264 | . 80595 | 1.14162 | . 90727 | 1.10220 | . 93961 | 1.06427 | 47 |
| 14 | . 84606 | 1.18194 | . 87646 | 1.14095 | . 90781 | 1.10156 | . 94016 | 1.06365 | 46 |
| 15 | . 84656 | 1.18125 | . 87698 | 1.14028 | . 90834 | 1.10091 | . 94071 | 1.06303 | 45 |
| 16 | . 84706 | 1.18055 | . 87749 | 1.13961 | . 90887 | 1.10027 | . 94125 | 1.06241 | 44 |
| 17 | . 84756 | 1.17986 | . 87801 | 1.13894 | . 90940 | 1.09963 | . 94180 | 1.06179 | 43 |
| 18 | . 84806 | 1.17916 | . 87852 | 1.13828 | . 00993 | 1.09899 | . 94235 | 1.06117 | 42 |
| 19 | . 84856 | 1.17846 | .87904 | 1.13761 | . 91046 | 1.09834 | . 94290 | 1.06056 | 41 |
| 20 | . 84906 | 1.17777 | . 87955 | 1.13694 | . 91099 | 1.09\%\% | . 94345 | 1.05994 | 40 |
| 21 | . 84956 | 1.17708 | .88007 | 1.13627 | . 91153 | 1.09706 | . 94400 | 1.05932 | 39 |
| 22 | . 85006 | 1.17638 | . 88009 | 1.13561 | . 91206 | 1.09642 | . 94455 | 1.058\% | 38 |
| 23 | . 85057 | 1.17569 | . 88110 | 1.13494 | . 91259 | 1.09578 | . 94510 | 1.05809 | 37 |
| 24 | . 85107 | 1.17500 | . 88162 | 1.13428 | . 91313 | 1.09514 | . 94565 | 1.05447 | 36 |
| 25 | . 85157 | 1.17430 | . 88214 | 1.13361 | . 91366 | 1.09450 | . 94620 | 1.05685 | 35 |
| 26 | . 85207 | 1.17361 | . 88265 | 1.13295 | . 91419 | 1.09386 | . 94676 | 1.05624 | 34 |
| 27 | . 85257 | 1.17292 | . 88317 | 1.13223 | . $914 \% 3$ | 1.09322 | . 94731 | 1.05562 | 33 |
| 28 | . 85308 | 1.17223 | . 88369 | 1.13162 | . 91526 | 1.09258 | . 94786 | 1.05501 | 32 |
| 29 | . 85358 | 1.17154 | . 88421 | 1.13096 | . 91580 | 1.09195 | . 94841 | 1.05439 | 31 |
| 30 | . 85408 | 1.17085 | 88473 | 1.13029 | . 91633 | 1.09131 | . 94896 | 1.05378 | 30 |
| 31 | . 85458 | 1.17016 | . 88524 | 1.12963 | . 91687 | 1.09067 | . 94952 | 1.05317 | 29 |
| 32 | . 85509 | 1.16947 | . 88576 | 1.12897 | . 91640 | 1.09003 | . 95007 | 1.05255 | 28 |
| 33 | . 85559 | 1.16878 | . 88628 | 1.12831 | . 91794 | 1.08940 | . 95062 | 1.05194 | 27 |
| 34 | . 85609 | 1.16809 | . 88680 | 1.12765 | . 91847 | 1.08876 | . 95118 | 1.05133 | 26 |
| 35 | . 85660 | 1.16741 | . 88732 | 1.12699 | . 91901 | 1.08813 | . 95173 | 1.05072 | 25 |
| 36 | . 85710 | 1.16672 | . 88784 | 1.12633 | . 91955 | 1.08749 | . 95229 | 1.05010 | 24 |
| 37 | . 85761 | 1.16603 | . 88836 | 1.12567 | . 22008 | 1.08686 | . 95284 | 1.04949 | 23 |
| 38 | . 85811 | 1.16535 | . 88888 | 1.12501 | . 92062 | 1.08622 | . 95340 | 1.04888 | 22 |
| 39 | . 85862 | 1.16466 | . 88940 | 1.12435 | . 22116 | 1.08559 | . 95395 | 1.04827 | 21 |
| 40 | . 85912 | 1.16398 | . 88992 | 1.12369 | . 92170 | 1.08496 | . 95451 | 1.04766 | 20 |
| 41 | . 85963 | 1.16329 | . 89045 | 1.12303 | . 92224 | 1.08432 | . 95506 | 1.04705 | 19 |
| 42 | . 86014 | 1.16261 | . 89097 | $1.12: 33$ | . 22224 | 1.08369 | . 95562 | 1.04644 | 18 |
| 43 | . 86064 | 1.16192 | . 89149 | $1.121 \%$ | . 92331 | 1.08306 | . 95618 | 1.04583 | 17 |
| 44 | . 86115 | 1.16124 | . 89201 | 1.12106 | . 92385 | 1.08243 | . 95673 | 1.04522 | 16 |
| 45 | . 86166 | 1.16056 | . 89253 | 1.12041 | . 92439 | 1.08179 | .95\%29 | 1.04461 | 15 |
| 46 | . 86216 | 1.15987 | . 89306 | 1.11975 | . 92493 | 1.08116 | . 95785 | 1.04401 | 14 |
| 47 | . 86267 | 1.15919 | . 89358 | 1.11909 | . 92547 | 1.08053 | . 95841 | 1.04340 | 13 |
| 48 | . 86318 | 1.15851 | . 89410 | 1.11844 | . 92601 | 1.07990 | . 95897 | 1.04279 | 12 |
| 49 | . 86368 | 1.15783 | . 89463 | 111778 | . 92655 | 1.07927 | . 95952 | 1.04218 | 11 |
| 50 | . 86419 | 1.15715 | . 89515 | 1.1 | . 92709 | 1.07864 | . 96008 | 1.04158 | 10 |
| 51 | . 86470 | 1.15647 | . 89567 | 1.11648 | . 92763 | 1.07801 | . 96064 | 1.04097 | 9 |
| 52 | . 86521 | 1.15579 | . 89620 | 1.11582 | . 92817 | 1.07738 | . 96120 | 1.04036 | 8 |
| 53 | . 86572 | 1.15511 | . 8969 | 1.11517 | . 92272 | 1.06676 | . 96176 | 1.03976 | 7 |
| 54 | . 86623 | 1.15443 | . 89725 | 1.11452 | . 22926 | 1.06613 | . 96232 | 1.03915 | 帾 |
| 55 | .86674 | 1.15375 | . 89777 | 1.11387 | . 92980 | 1.07550 | . 96288 | 1.03855 | 5 |
| 56 | .867\% | 1.15308 | . 89830 | 1.11321 | . 93034 | 1.0\%487 | . 96344 | 1.03794 | 4 |
| 57 | . 86776 | 1.15240 | . 89883 | 1.11256 | . 93088 | 1.07425 | . 98400 | 1.03734 | 3 |
| 58 | . 86827 | 1.15172 | . 89935 | 1.11191 | . 93143 | 1.07362 | . 96457 | 1.036 4 | 2 |
| 59 | . 868878 | 1.15104 | . 89938 | 1.11126 | . 93197 | 1.07799 | . 96513 | 1.03613 | 1 |
| $\underline{60}$ | . 86929 | 1.15037 | . 90040 | 1.11061 | 93252 | 1.07237 | 96569 | 1.03553 | 0 |
|  | Cotang | Tang | Cotang | Tang | Cotang | Tang | Cotang | Tang |  |
|  |  | - |  | ${ }^{\circ}$ |  |  | 4 |  |  |

TABLE VII.-Continued.
Natural Tangents and Cotangents.

|  | $44^{\circ}$ |  |  |  | $44^{\circ}$ |  |  |  | $44^{\circ}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Tang | Cotang |  |  | Tang | Cotang |  |  | Tang | Cotang |  |
| 0 | . 96.569 | 1.035553 | 60 | 20 | . 97700 | 1.02355 | 40 | 40 | . 98843 | 1.01170 | 20 |
| 1 | . 96625 | 1.03493 | 59 | 21 | . 97756 | $1.02 \% 295$ | 39 | 41 | . 98901 | 1.01112 | 19 |
| 2 | . 96681 | 1.03433 | 58 | 22 | .97813 | 1.022336 | 38 | 42 | . 98958 | 101053 | 18 |
| 3 | . 96738 | 1.033372 | 57 | 23 | .97870 | 1.02176 | 37 | 43 | . 99016 | 1.00994 | 17 |
| 4 | . 93791 | 1.03312 | 56 | 24 | .97927 | 1.02117 | 36 | 44 | . 99073 | 1.00935 | 16 |
| 5 | .963;0 | 1.03252 | 55 | 25 | . 97984 | 1.02057 | 35 | 45 | . 99131 | 1.00876 | 15 |
| 6 | . 96907 | 1.03192 | 54 | 26 | . 98041 | 1.01998 | 34 | 46 | . 99189 | 1.00818 | 14 |
| 7 | . 96963 | 1.03132 | 53 | 27 | . 98098 | 1.01939 | 33 | 47 | . 99247 | 1.00759 | 13 |
| 8 | . 97020 | 1.03072 | 53 | 28 | . 98155 | 1.01879 | 32 | 48 | . 99304 | 1.00701 | 12 |
| 9 | .970¢6 | 1.03012 | 51 | 29 | . 98213 | 1.01820 | 31 | 49 | . 99362 | 1.00642 | 11 |
| 10 | . 9 \%133 | 1.02952 | 50 | 30 | .982\% | 1.01761 | 30 | 50 | . 99420 | 1.00583 | 10 |
| 11 | . 97189 | 1.02892 | 49 | 31 | . 98327 | 1.01702 | 29 | 51 | . 99478 | 1.00525 | 9 |
| 12 | . 97246 | 1.02832 | 48 | 32 | . 98384 | 1.01642 | 28 | 52 | . 99536 | 1.00467 |  |
| 13 | . 97302 | 1.02772 | 47 | 33 | . 98441 | 1.01583 | 27 | 53 | . 99594 | 1.00408 | 7 |
| 14 | . 97359 | 1.02713 | 46 | 34 | . 98499 | 1.01524 | 26 | 54 | . 99652 | 1.00350 | 6 |
| 15 | . 97416 | 1.02653 | 45 | 35 | . 98556 | 1.01465 | 25 | 55 | . 99710 | 1.00291 | 5 |
| 16 | . 97472 | 1.02593 | 44 | 36 | . 98613 | 1.01406 | 24 | 56 | . 99768 | 1.00233 | 4 |
| 17 | . 97529 | 1.02533 | 43 | 37 | . 98671 | 1.01347 | 23 | 57 | . 99886 | 1.00175 | 3 |
| 18 | . 97586 | 1.02474 | 42 | 38 | . 98728 | 1.01288 | 22 | 58 | . 99884 | 1.00116 | 2 |
| 19 | . 97643 | 1.02414 | 41 | 39 | . 98786 | 1.01229 | 21 | 59 | . 99942 | 1.00058 | 1 |
| 20 | .97700 | 1.02355 | 40 | 40 | . 98843 | 1.01170 | 20 | 60 | 1.00000 | 1.00000 | 0 |
|  | Cotang Tang |  |  | , | Cotang Tang |  |  | , | Cotang Tang |  |  |
|  | $45^{\circ}$ |  |  |  | $45^{\circ}$ |  |  |  | $45^{\circ}$ |  |  |



| $z^{2 l}$ | $8^{262 \cdot 0}$ | $z \cdot 6 \angle t$ | $1 \cdot \downarrow z$ ¢ | (0u15to) soou | ts.tt | 98914 | 86884 | zโย์ | 801.69 | $\bigcirc \mathrm{C}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $z^{u}$ | Loos.0 | $0 \cdot{ }^{\circ}$ | $z \cdot 6 z S$ |  | 18.0 t | $S_{\text {Igt }}$ | 10918 | ¢ $\angle \varsigma \varepsilon$ | \$80.69 | c8 ${ }^{\text {b }}$ |
| $z^{u}$ | zzoE.0 | $2 \cdot 98{ }^{\text {b }}$ | L.1ES |  | Er $\cdot 8{ }^{\text {b }}$ | EStLL | tolt 8 | $\varepsilon \varepsilon_{8} \varepsilon$ | $090 \cdot 69$ | -9 ${ }^{\text {b }}$ |
| $z^{u}$ | zzoE.0 | $z \cdot 9{ }^{\text {b }}$ | L.IES | (ouzot*o) sos u | $\varepsilon_{8}{ }^{6} 6$ | L6108 | tolls | $011 t$ | $980 \cdot 69$ | -tt |
| $z^{u}$ | 900\&.0 | $8^{-\varepsilon} 8^{\text {b }}$ | 0.6zS | (02098.0) sou $u$ | $8^{t \cdot 15}$ | ot 8 z8 | 96506 | gott | $110 \cdot 69$ | - $\square^{\square}$ |
| $z^{u}$ | 9<6z*o | - $06 \angle$ | S-SzS |  | 90.ES | $\varepsilon_{8} \varepsilon^{\text {¢ }} 8$ | LLEE6 | $6 z \angle b$ | L86.89 | $\bigcirc{ }^{\circ 1}$ |
| $z^{24}$ | z\&6z*o | $8 \cdot 1 /{ }^{\circ}$ | 0.915 |  | LS.ts | こと8 ${ }^{\text {c }}$ | ttog6 | 6 LoS | †96.89 | ${ }_{08} \mathcal{E}^{1}$ |
| $z^{u}$ | £ $\angle 8 \%^{\circ} \circ$ |  | L.Sos | (ouLEE.0) sou $u$ | zo.9S | ES 106 | £6886 | 19tS | 1t6.89 | -9£ |
| $8^{u}$ | -08z*o | L-OSb | o.86t | ( $0 \sim 02 \varepsilon \cdot 0$ ) Sov u | -t. 25 | £LEz6 | zzoror | 1885 | 816.89 | $\bigcirc \downarrow \mathcal{L}$ |
| $z^{u}$ | Silz'o | $8.98 t$ | 8. $21+$ |  | 14.85 | $18{ }^{\text {tot6 }}$ | LzEEOr | $8^{\dagger} \varepsilon_{9}$ | L68.89 | ๐ $\sim$ E |
| $z^{u}$ | L1920 | - ¢ı ${ }^{\text {t }}$ | -09 ${ }^{\text {b }}$ | (0u88**) sos u | S6.65 | 9<t96 | LoSSor | 6989 | ¢ 4889 |  |
| - $^{\text {-3x }}$ |  | *วมว ${ }^{\text {N }}$ uI | ${ }^{\text {spre }}{ }_{\text {A }} \mathrm{u}^{\text {I }}$ | ${ }^{-1010}{ }^{\text {a }}$ | -so! ${ }^{\text {d }}$ uI |  | ${ }^{\text {spre }}{ }_{\text {A }}$ uI | -sว!w |  |  |
|  |  |  |  |  |  |  |  | -urel јo эp!S јо чน누วт |  | - |


'IIIA GTGVL
TABLE IX．

|  |  | - | $0 \times \infty$ |  | くれやかO Qvivivio | ＊$\infty$ Q ${ }^{\circ}$ ตッズメン |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | \％ |  | OOMM థ్లైల్లైల్ల | ヘnल๐ a <br>  | －○○のに <br>  | －ロNAN <br>  |
|  | 잉 | －MMA～ <br>  |  |  <br> 率安的以 | ヘッササ～ ทํํ우웅 | かのいへ 8ธ்ต்ல்ல் |
|  | 19 <br> 8. <br> 8 | $00+7+$ <br>  |  | ～ッ Mo n <br>  | ロッ～мm ถ๐๐ㅇํํㅜㅗ | ○ $\quad$ 毋 0 ○ ベウベペ |
|  |  |  | に 000 に 4in ig |  |  <br>  | ○ーのッм <br>  |
|  | O \％ O． |  ผ่ บ่ํํํํ | のはNへに ヘ่ถ่ํํํ | ッロッッタ <br> さべがが | mo O N m がゅめかの | ーヘッド <br>  |
|  | \％ | $\bigcirc \times+\infty$ <br>  | ＋Mono <br> 人完がが | ＋～no＋ <br>  |  | $\cdots \infty$ mo $a$ <br> 우우ㅇㅜㅜㅜㅜ |
|  | $\stackrel{10}{0}$ | Nmotr ทั่ ทั๗ | $\infty$ ○○○．．． | คヘッMm <br>  | －$+\infty$ ○ <br> $\dot{\sim}$ <br>  |  |
|  | $\stackrel{9}{0}$ | ON No No N | மio $\dot{\circ}$ 우쑤쑤 | + ヘット～ <br>  | mam＾a స్ల్లెల్లైల్ల |  |
|  | $\stackrel{\text { Q }}{\text { ¢ }}$ | $\rightarrow-\infty$ <br>  |  |  |  |  |
|  | $\stackrel{-1}{0}$ |  |  |  | ตo ヘisioutio |  |
|  | 응 | $\infty \rightarrow \infty$ no <br>  |  | +00 mo <br>  |  ตnํㅜㄴ <br>  | Nomm． <br>  |
|  | O |  |  |  |  |  |
| ¢ |  |  | $0 \geq \infty, 0$ |  | Q $\# 10 \infty 0$ <br>  | $+\infty \mathrm{CNO}$ <br>  |



TABLE XI.
Volumes by the Prismoidal Formula. § 320.

| $\begin{aligned} & \text { 䔍 } \\ & 0 \\ & 0 \end{aligned}$ | Heights. |  |  |  |  |  |  |  |  |  | Corrections for tenths in height. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |
| 1 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | . 1 | 0 |
| 2 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 6 | 6 | . 2 | 0 |
| 3 | 1 | 2 | 3 | 4 | 5 | 6 | 6 | 7 | 8 | 9 | . 3 | 0 |
| 4 | 1 | 2 | 4 | 5 | ${ }_{8}$ | 7 | 9 | 10 | 11 | 12 | . 4 | 1 |
| 5 | -2 | -3 | -5 | -6 | -8 | -9 | $-11$ | -12 | -14 | -15 | . 5 | 1 |
| 6 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 | 19 | . 6 | 1 |
| 7 | 2 | 4 | ${ }_{7}^{6}$ | 9 | 11 | 13 | 15 | 17 | 19 | 22 | . 7 | 1 |
| 8 | $\stackrel{2}{2}$ | 5 | 7 | 10 | 12 | 15 | 17 | 20 | 22 | 25 | . 8 | 1 |
| 9 | 3 | 6 | 8 | 11 | 14 | 17 | 19 | 22 | 25 | 28 | . 9 |  |
| 10 | 3 | 6 | 9 | 12 | 15 | 19 | 22 | 25 | 28 | 31 | . 9 |  |
| 11 | 3 | 7 | 10 | 14 | 17 | 20 | 24 | 27 | 31 | 34 | . 1 |  |
| 12 | 4 | 7 | 11 | 15 | 19 | 22 | 26 | 30 | 33 | $3 \pi$ | . 2 | 1 |
| 13 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | . 3 | 1 |
| 14 | 4 | 9 | 13 | 17 | 22 | 26 | 30 | 35 | 39 | 43 | . 4 |  |
| 15 | -5 | -9 | -14 | -19 | $-23$ | -28 | $-32$ | -37 | -42 | -46 | . 5 | 2 |
| 16 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 44 | 49 | . 6 | 3 |
| 17 | 5 | 10 | 16 | 21 | 26 | 31 | 37 | 42 | 47 | 52 | . 7 | 3 |
| 18 | 6 | 11 | 17 | 22 | 28 | 33 | 39 | 44 | 50 | 56 | . 8 | 4 |
| 19 | 6 | 12 | 18 | 23 | 29 | 35 | 41 | 4 | 53 | 59 | . 9 |  |
| 20 | 6 | 12 | 19 | 25 | 31 | 37 | 43 | 49 | 56 | 62 |  |  |
| 21 | 7 | 13 | 19 | ${ }_{26}^{26}$ | 32 | 39 | 45 | 52 | 58 | 65 | - | , |
| 22 | 7 | 14 | 20 | 27 | 34 | 41 | 48 | 54 | 61 | 68 | . 2 |  |
| 23 | 7 | 14 | 21 | 28 | - 35 | 43 | 50 | 57 | 64 | 71 | . 3 | 2 |
| 24 | 7 | 15 | 22 | 30 | ${ }^{37}$ | 44 | 52 | 59 | 67 | I4 | . 4 |  |
| 25 | -8 | $-15$ | $-23$ | -31 | $-39$ | -46 | -54 | -62 | -69 | -77 | - 5 | 4 |
| ${ }_{27}^{26}$ | 8 | 16 | 24 | 32 <br> 33 | 40 | 48 | 56 58 | 64 | ${ }^{7}$ | 80 | . 6 | 5 |
| 27 | 8 | 17 | 25 | 33 | 42 | 50 | 58 | 67 | 75 | 83 | . 7 | 5 |
| 28 | 9 | 17 | 26 | 35 | 43 | 52 | 60 | 69 | 78 | 86 | . 8 | 6 |
| 29 | 9 | 18 | 27 | $3{ }^{36}$ | 45 | 54 | 63 | \% | 81 | 90 | . 9 |  |
| 80 | 9 | 19 | 28 | 37 | 46 | 56 | 65 | 74 | 83 | 93 |  |  |
| 31 | 10 | 19 | 29 | 38 | 48 | 57 | 67 | 77 | 86 | 96 | . 1 |  |
| 32 | 10 | 20 | 30 | 40 | 49 | 59 | 69 | \%9 | 89 | 99 | . 2 | 2 |
| 33 | 10 | 20 | 31 | 41 | 51 | 61 | 71 | 81 | 92 | 102 | . 3 | 3 |
| 34 | 10 | 21 | 21 | 42 | 52 | 63 | 73 | 84 | 94 | 105 | . 4 |  |
| 35 | -11 | -22 | $-32$ | -43 | $-54$ | -65 | -76 | -86 | -97 | -108 | - 5 | 5 |
| 36 | 11 | 22 | 33 | 44 | 56 | 67 | 78 | 89 | 100 | 111 | . 6 |  |
| 37 | 11 | 23 | 31 | 46 | 57 | 69 | 80 | 91 | 103 | 114 | . 7 |  |
| 38 | 12 | 23 | 3.5 | 47 | 59 | 70 | 82 | 94 | 106 | 11\% | . 8 |  |
| 39 | 12 | 24 | 36 | 48 | 60 | \%2 | 84 | 96 | 108 | 120 | . 9 | 10 |
| 40 | 12 | 25 | 37 | 49 | 62 | 74 | 86 | 99 | 111 | 123 |  |  |
| 41 | 13 | 25 | 88 | 51 | 63 | 76 | 89 | 101 | 114 | 127 | . 1 |  |
| 42 | 13 | 26 | 39 | 5. | 65 | 78 | 91 | 104 | 117 | 130 | . 2 | 3 |
| 43 | 13 | 27 | 40 | 53 | 66 | 80 | - 93 | 106 | 119 | 133 | . 3 |  |
| 44 | -14 | 27 -28 | 41 -42 | 54 -56 | 68 -69 | 81 -83 | 95 -97 | -109 | 129 | 136 | . 4 | ${ }_{\sim}^{6}$ |
| 46 | -14 | -28 | -43 | -56 | -69 | -83 85 | -97 99 | -111 | -125 128 | -139 | . 5 |  |
| 47 | 15 | 29 | 44 | 58 | 7 | 87 | 102 | 116 | 131 | 145 | . 7 | 10 |
| 48 | 15 | 30 | 44 | 59 | 74 | 89 | 104 | 119 | 133 | 148 | . 8 | 11 |
| 49 | 15 | 30 | 45 | 60 | 76 | 91 | 106 | 121 | 136 | 151 |  |  |
| 50 | 15 | 31 | 46 | 62 | 77 | 93 | 108 | 123 | 139 | 154 |  |  |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |
|  | . 1 | . 2 | $\cdot 3$ | 4 | . 5 | . 6 | . 7 | . 8 | $\cdot 9$ | Corrections for tenths in width. |  |  |
|  | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |  |  |  |  |  |

TABLE XI.-Continued.
Volumes by the Prismoidal Formula.

| $\begin{aligned} & \text { 㡙 } \\ & \stackrel{0}{0} \end{aligned}$ | Heights. |  |  |  |  |  |  |  |  |  | Corrections for tenths in height. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |
| 51 | 16 | 31 | 47 | 63 | 79 | 94 | 110 | 126 | 142 | 157 | . 1 | 2 |
| 52 | 16 | 32 | 48 | 64 | 80 | 96 | 112 | 128 | 144 | 160 | . 2 | 3 |
| 53 | 16 | 33 | 49 | 65 | 82 | 98 | 115 | 131 | 147 | 163 | $\cdot 3$ | 5 |
| 54 | 17 | 33 | 50 | 67 | 83 | 100 | 117 | 133 | 150 | 167 | . 4 | 7 |
| 55 | $-17$ | $-34$ | $-51$ | -68 | -85 | -102 | -119 | -136 | -153 | -170 | . 5 | 8 |
| 56 | 17 | 35 | 52 | 69 | 86 | 104 | 121 | 138 | 156 | 173 | . 6 | 10 |
| 57 | 18 | 35 | 53 | 70 | 88 | 106 | 123 | 141 | 158 | -176 | . 7 | 12 |
| 58 | 18 | 36 | 54 | 72 | 90 | 107 | 125 | 143 | 161 | 179 | . 8 | 14 |
| 59 | 18 | 36 | 55 | 73 | 91 | 109 | 127 | 146 | 164 | 182 | . 9 | 15 |
| 60 | 19 | 37 | 56 | 74 | 93 | 111 | 130 | 148 | 167 | 185 |  |  |
| 61 | 19 | 38 | 56 | 75 | 94 | 113 | 132 | 151 | 169 | 188 | . 1 | 2 |
| 62 | 19 | 38 | 57 | 77 | 96 | 115 | 134 | 153 | 172 | 191 | . 2 | 4 |
| 63 | 19 | 39 | 58 | 78 | 97 | 117 | 136 | 156 | 175 | 194 | $\cdot 3$ |  |
| 64 | 20 | 40 | 59 | 79 | 99 | 119 | 138 | 158 | 178 | 197 | . 4 |  |
| 65 | $-20$ | -40 | -60 | -80 | -100 | -120 | -140 | -160 | -181 | -201 | . 5 | 10 |
| 66 | 20 | 41 | 61 | 81 | 102 | 122 | 143 | 163 | 183 | 204 | . 6 | 12 |
| 67 | 21 | 41 | 62 | 83 | 103 | 124 | 145 | 165 | 186 | 207 | . 7 | 14 |
| 68 | 21 | 42 | 63 | 84 | 105 | 126 | 147 | 168 | 189 | 210 | . 8 | 16 |
| 69 | 21 | 43 | 64 | 85 | 106 | 128 | 149 | 170 | 192 | 213 | . 9 | 18 |
| 70 | 22 | 43 | 65 | 86 | 108 | 130 | 151 | 173 | 194 | 216 | . 9 |  |
| 71 | 22 | 44 | 66 | 88 | 100 | 131 | 153 | 175 | 197 | 219 | . 1 | 2 |
| 72 | 22 | 44 | 67 | 89 | 111 | 133 | 156 | 178 | 200 | 222 | .2 | 5 |
| 73 | 23 | 45 | 68 | 90 | 113 | 135 | 158 | 180 | 203 | 225 | $\cdot .3$ | 7 |
| 74 | 23 | 46 | 69 | 91 | 114 | 137 | 160 | 183 | 206 | 228 | . 4 | 9 |
| 75 | -23 | -46 | -69 | -93 | -116 | -139 | -162 | -185 | -208 | $-231$ | . 5 | 12 |
| 76 | 23 | 47 | 70 | 94 | 117 | 141 | 164 | 188 | 211 | 235 | . 6 | 14 |
| 77 | 24 | 48 | 71 | 95 | 119 | 143 | 166 | 190 | 214 | 238 | . 7 | 16 |
| 78 | 24 | 48 | 72 | 96 | 120 | 144 | 169 | 193 | 217 | 241 | . 8 | 19 |
| 79 | 24 | 49 | 73 | 98 | 122 | 146 | 171 | 195 | 219 | 244 | . 9 | 21 |
| 80 | 25 | 49 | 74 | 99 | 123 | 148 | 173 | 198 | 222 | 247 |  |  |
| 81 | 25 | 50 | 75 | 100 | 125 | 150 | 175 | 200 | 225 | 250 | . 1 | 8 |
| 82 | 25 | 51 | 76 | 101 | 127 | 152 | 177 | 202 | 228 | 253 | . 2 | 5 |
| 83 | 26 | 51 | 77 | 102 | 128 | 154 | 179 | 205 | 231 | 256 | $\cdot 3$ | 8 |
| 84 | 26 | 52 | 78 | 104 | 130 | 156 | 181 | 207 | 233 | 259 | . 4 |  |
| 85 | -26 | $-52$ | -79 | -105 | -131 | -157 | -184 | -210 | -236 | -262 | . 5 | 13 |
| 86 | 27 27 | 5 | 80 | 107 | 133 <br> 134 | 159 161 | 186 188 | 212 215 | 239 242 | 265 269 | .6 | 16 |
| 88 | 27 | 54 | 81 | 109 | 136 | 163 | 190 | 217 | 244 | 272 | . 8 | 21 |
| 89 | $\stackrel{27}{ }$ | 55 | 82 | 110 | 137 | 165 | 192 | 220 | 247 | 275 | . 9 | 24 |
| 90 | 28 | 56 | 83 | 111 | 139 | 167 | 194 | 222 | 250 | 278 |  |  |
| 91 | 28 | 56 | 84 | 112 | 140 | 169 | 197 | 225 | 253 | 281 | . 1 | 8 |
| 92 | 28 | 57 | 85 | 114 | 142 | 170 | 199 | 227 | 256 | 284 | . 2 | 6 |
| 93 | 29 | 57 | 86 | 115 | 144 | 172 | 201 | 230 | 258 | 287 | .3 | 9 |
| 94 | 29 | 58 | 87 | 116 | 145 | 174 | 203 | 232 | 261 | 290 | . 4 | 12 |
| 95 | -29 | -59 -59 | -88 -89 | -117 | -147 | $-176$ | -205 | -235 | -264 | -293 |  | 15 |
| 96 97 | 30 30 | 59 60 | 89 90 | 119 120 | 148 | 178 | 207 | 237 | 267 | 296 | . 6 | 18 |
| 98 | 30 | 60 | 90 | 120 | 150 | 180 181 | 210 212 | 240 242 | 269 272 | $\stackrel{299}{ }$ | .7 | ${ }_{23}^{21}$ |
| 99 | 31 | 61 | 92 | 122 | 153 | 183 | 214 | 244 | $2{ }_{2} 2$ | 302 306 |  |  |
| 100 | 31 | 62 | 93 | 123 | 154 | 185 | 216 | 247 | 278 | 309 |  |  |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |
|  | . 1 | . 2 | $\cdot 3$ | . 4 | . 5 | . 6 | -7 | . 8 | $\cdot 9$ | Corrections for tenths in width. |  |  |
|  | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |  |  |  |  |  |

TABLE XI.-Continued.
Volumes by the Prismoidal Formula.

| $\begin{aligned} & \text { 号 } \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | Heights. |  |  |  |  |  |  |  |  |  | Corrections for tenths in height. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |  |  |
| 1 | 3 | 4 | 4 | 4 | 5 | 5 | 5 | ${ }^{6}$ |  | 6 | . 1 | 0 |
| 2 | 7 | 7 | 8 |  | 9 | 10 | 10 | 11 | 12 | 12 | .2 | 0 |
| 3 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | $\cdot 3$ | 0 |
| 4 | 14 | 15 | 16 | 17 | 19 | 20 | 21 | 22 | -23 | -25 | - 4 | 1 |
| 5 | $-17$ | -19 | -20 | -22 | -23 | -25 | -26 | -28 | -29 | -31 | . 5 | 1 |
| 6 | 20 | ${ }_{28}^{22}$ | 24 | 26 | 28 | 30 | 31 | 33 | 35 | 37 | . 6 |  |
| 7 | 24 | 26 | 28 | 30 | 32 | 35 | 37 | 39 | 41 | 43 | .7 |  |
| 8 | 27 | 30 | 32 | 35 | 37 | 40 | 42 | 44 | 47 | 49 | . 8 |  |
| 9 | 31 | 33 | 36 | 39 | 42 | 44 | 47 | 50 | 53 | 56 | $\cdot 9$ | 1 |
| 10 | 34 | 37 | 40 | 43 | 46 | 49 | 52 | 56 | 59 | 62 |  |  |
| 11 | 37 | 41 | 44 | 48 | 51 | 54 | 58 | 61 | 65 | 68 | . 1 | 0 |
| 12 | 41 | 44 | 48 | 52 | 56 | 59 | 63 | 67 | 70 | 74 | . 2 | 1 |
| 13 | 44 | 48 | 52 | 56 | 60 | 64 | 68 | 77 | 76 | 80 | $\cdot 3$ | 1 |
| 14 | 48 | 52 | 56 | 60 | 65 | 69 | 73 | 78 | 82 | 86 | . 4 | 2 |
| 15 | -51 | $-56$ | -60 | -65 | -69 | -74 | -79 | -83 | -88 | -93 | -5. | 2 |
| 16 | 54 | 59 | 64 | 69 | 74 | \%9 | 84 | 89 | 94 | 99 | . 6 | 3 |
| 17 | 58 | 63 | 68 | 73 | 79 | 84 | 89 | 94 | 100 | 105 | .7 | 3 |
| 18 | 61 | 67 | 72 | 78 | 83 | 89 | 94 | 100 | 106 | 111 | . 8 | 4 |
| 19 | 65 | 70 | 76 | 82 | 88 | 94 | 100 | 106 | 111 | 117 | . 9 | 4 |
| 20 | 68 | 74 | 80 | 86 | 93 | 99 | 105 | 111 | 117 | 123 |  |  |
| 21 | 71 | 78 | 84 | 91 | 97 | 104 | 110 | 117 | 123 | 130 | . 1 | 1 |
| 22 | 75 | 81 | 88 | 95 | 102 | 109 | 115 | 122 | 129 | 136 | .2 | 2 |
| 23 | 78 | 85 | 92 | 99 | 106 | 114 | 121 | 128 | 135 | 142 | $\cdot 3$ | 2 |
| 24 | 81 | 89 | 96 | 104 | 111 | 119 | 126 | 133 | -141 | . 148 | . 4 |  |
| 25 | -85 | -93 | -100 | -108 | -116 | $-123$ | -131 | -139 | -147 | -154 | .5 | 4 |
| $\stackrel{26}{27}$ | 88 | 96 100 | 104 | 112 | 120 | 128 | 136 142 | 144 150 | 152 158 | 160 167 | . 6 | 5 |
| 28 | 95 | 104 | 112 | 121 | 130 | 138 | 147 | 156 | 164 | 173 | . 88 | ${ }_{6}$ |
| 29 | 98 | $10 \%$ | 116 | 125 | 134 | 143 | 152 | 161 | 170 | 179 | . 9 | 7 |
| 30 | 102 | 111 | 120 | 130 | 139 | 148 | $15 \%$ | 167 | 176 | 185 |  |  |
| 31 | 105 | 115 | 124 | 134 | 144 | 153 | 163 | 172 | 182 | 191 | . 1 | 1 |
| 32 | 109 | 119 | 128 | 138 | 148 | 158 | 168 | 178 | 188 | 198 | . 2 | 2 |
| 33 | 112 | 122 | 132 | 143 | 153 | 163 | 173 | 183 | 194 | 204 | $\cdot 3$ | 3 |
| 34 | 115 | 126 | 136 | 147 | 157 | 168 | 178 | 189 | 199 | 210 | . 4 |  |
| 35 | -119 | -130 | -140 | -151 | $-162$ | -173 | -184 | -194 | -205 | -216 |  |  |
| 36 | 122 | 133 | 144 | 156 | 167 | 178 | 189 | 200 | 211 | 222 | . 6 | 6 |
| 37 | 126 | 137 | 148 | 160 | 171 | 183 | 194 | 206 | 217 | 228 | . 7 | 8 |
| 38 | 129 | 141 | 152 | 164 | 176 | 188 | 199 | 211 | 223 | 235 | . 8 |  |
| 39 | 132 | 144 | 156 | 169 | 181 | 193 | 205 | 217 | 229 | 241 | $\cdot 9$ | 10 |
| 40 | 136 | 148 | 160 | 173 | 185 | 198 | 210 | 222 | 235 | 247 |  |  |
| 41 | 139 | 152 | 165 | $17 \%$ | 190 | 202 | 215 | 228 | 240 | 253 | . 1 | 1 |
| 42 | 143 | 156 | 169 | 181 | 194 | 207 | 220 | 233 | 246 | 259 | . 2 | 3 |
| 43 | 146 | 159 | 173 | 186 | 199 | 212 | 226 | 239 | 252 | 265 | $\cdot 3$ | 4 |
| 44 | 149 | 163 | 177 | 190 | 204 | 217 | 231 | 244 | 258 | ${ }_{2 \uparrow}^{27}$ | -4 |  |
| 45 | -153 | $-167$ | -181 | -194 | -208 | -222 | -236 | -250 | -264 | -278 | . 5 |  |
| 46 | 156 | 170 | 185 | 199 | 213 | 227 | 241 | 256 | 270 | 284 | . 6 | 8 |
| 47 | 160 | 174 | 189 | 203 | 218 | 232 | 247 | $\stackrel{261}{ }$ | 276 | 290 | .7 | 10 |
| 48 | 163 | 178 | 193 | 207 | 222 | 237 | ${ }_{257}^{252}$ | 267 | $\stackrel{281}{281}$ | 296 | . 8 | 11 |
| 49 | 166 | 181 | 197 | 212 | 227 | 242 | 257 | $\underset{278}{272}$ | $\stackrel{287}{28}$ | 3 | $\cdot 9$ |  |
| 50 | 170 | 185 | 201 | 216 | 231 | 247 | 262 | 278 | 293 | 309 |  |  |
|  | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |  |  |
|  | . 1 | . 2 | . 3 | . 4 | . 5 | . 6 | $\cdot 7$ | . 8 | . 9 |  |  |  |
|  | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |  |  |  |

TABLE XI.-Continued.
Volumes by the Prismoidal Formula.

| $\begin{aligned} & \text { 号 } \\ & \frac{0}{0} \\ & \hline \end{aligned}$ | Heights. |  |  |  |  |  |  |  |  |  | Corrections for tenths in height. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |  |  |
| 51 | 173 | 189 | 205 | 220 | 236 | 252 | 268 | 283 | 299 | 315 | . 1 | 2 |
| 52 | 177 | 193 | 209 | 225 | 241 | 257 | 273 | 289 | 305 | 321 | . 2 | 3 |
| 53 | 180 | 196 | 213 | 229 | 245 | 262 | 278 | 294 | 311 | 327 | . 3 | 5 |
| 54 | 183 | 200 | 217 | 233 | 250 | 267 | 283 | 200 | 317 | 333 | . 4 | 7 |
| 55 | -187 | -204 | -221 | -238 | -255 | $-2 \% \%$ | $-289$ | $-306$ | -323 | -340 | . 5 |  |
| 56 | 190 | 207 | 225 | 242 | 259 | $2{ }^{27}$ | 294 | 311 | 328 | 346 | . 6 | 10 |
| 57 | 194 | 211 | 229 | 246 | 264 | 281 | 299 | 317 | 334 | 352 | .7 | 12 |
| 58 | 197 | 215 | 233 | 251 | 269 | 286 | 304 | 322 | 340 | 358 | . 8 | 14 |
| 59 | 200 | 219 | 237 | 255 | ${ }^{273}$ | 291 | 310 | 328 | 346 | 364 | .9 | 15 |
| 60 | 204 | 222 | 241 | 259 | 278 | 296 | 315 | 333 | 352 | $2 \% 0$ |  |  |
| 61 | 207 | 226 | 245 | 264 | 282 | 301 | 320 | 339 | 358 | 377 | . 1 | 2 |
| 62 | 210 | 230 | 249 | 268 | 287 | 306 | 325 | 344 | 364 | 383 | .2 | 4 |
| 63 | 214 | 233 | 253 | 27\% | 292 | 311 | 331 | 350 | 369 | 389 | $\cdot 3$ | 6 |
| 64 | 217 | 237 | 257 | 277 | 296 | 316 | 336 | 356 | $3{ }^{375}$ | 395 | . 4 | 8 |
| 65 | -221 | -241 | -261 | -281 | -301 | -321 | -341 | -361 | $-381$ | -401 | . 5 | 10 |
| 66 | 224 | 244 | 265 | 285 | 306 | 326 | 346 | 367 | 387 | 407 | . 6 | 12 |
| 67 | 227 | 248 | 269 | 290 | 310 | 331 | 352 | 372 | 393 | 414 | .7 | 14 |
| 68 | 231 | 252 | 273 | 294 | 315 | 336 | 357 | 378 | 399 | 420 | . 8 | 16 |
| 69 | 234 | 256 | 274 | 298 | 319 | 341 | 362 | 383 | 405 | 426 | . 9 | 18 |
| 70 | 238 | 259 | 281 | 302 | 324 | 346 | 367 | 389 | 410 | 432 |  |  |
| 71 | 241 | 263 | 285 | 307 | 329 | 351 | 373 | 394 | 416 | 438 | . 1 | 2 |
| 72 | 244 | 267 | 289 | 311 | 333 | 356 | 378 | 400 | 422 | 444 | .2 | 5 |
| 73 | 248 | 270 | 293 | 315 | 338 | 360 | 383 | 406 | 428 | 451 | .3 | 7 |
| 74 | 251 | 274 | 297 | 320 | 343 | 36.5 | 388 | 411 | 434 | 457 | . 4 | 9 |
| 75 | -255 | -278 | -301 | -324 | -347 | -3i0 | -394 | -417 | -140 | -463 | $\cdot 5$ | 12 |
| 76 | 258 | 281 | 305 | $3 \geqslant 8$ | 335 | 375 | 399 | 422 | 446 | 469 | . 6 | 14 |
| 77 | 261 | 285 | 309 | 333 | 356 | 380 | $40^{\circ}$ | 428 | 452 | 475 | $\cdot 7$ | 16 |
| 78 | 265 | 289 | 313 | 337 | 361 | 385 | 409 | 433 | 457 | 481 | . 8 | 19 |
| 79 | 268 | 293 | 317 | 341 | 366 | 390 | 415 | 439 | 463 | 488 | . 9 | 21 |
| 80 | 272 | 296 | 321 | 346 | $3 \% 0$ | 395 | 420 | 444 | 469 | 494 |  |  |
| 81 | 275 | 300 | 325 | 350 | 375 | 400 | 425 | 450 | 475 | 500 | . 1 | 3 |
| 82 | 278 | 304 | 329 | 354 | 380 | 405 | 430 | 456 | 481 | 506 | . 2 | 5 |
| 83 | 282 | 307 | 333 | 359 | 384 | 410 | 435 | 461 | 487 | 512 | $\cdot 3$ | 8 |
| 84 | 285 | 311 | 337 | 363 | 389 | 415 | 441 | 467 | 493 | 519 | . 4 | 10 |
| 85 | -289 | -315 | -341 | $-367$ | -394 | -420 | -446 | $-4 \% 2$ | -498 | -525 |  | 13 |
| 86 | 292 | ${ }^{319}$ | 345 | ${ }^{372}$ | 398 | 425 | 451 | $4 \% 8$ | 504 | 531 | . 6 | 16 |
| 87 | 29.5 | 322 | 349 | 376 | 403 | 430 | 456 | 483 | 510 | 537 |  | 18 |
| 88 | 299 | 326 | 353 | 380 | 407 | 435 | 462 | 489 | 516 | 543 | . 8 | 21 |
| 89 | 303 | 3.30 | ${ }^{357}$ | 385 | 412 | 440 | 467 | 494 | 522 | 549 | . 9 | 24 |
| 90 | 306 | 333 | 361 | 389 | 417 | 444 | $4{ }^{4}$ | 500 | 528 | 556 |  |  |
| 91 | 309 | 337 | 365 | 393 | 421 | 449 | 477 | 506 | - 534 | 562 | . 1 | 3 |
| 92 | 312 | 341 | 369 | 398 | 426 | 454 | 483 | 511 | 540 | 568 | . 2 | 6 |
| 93 | 316 | 344 | 373 377 | 402 | 431 | 459 | 488 | 517 | 545 | 574 | $\cdot 3$ | 9 |
| 94 | 319 | 348 | 377 | 406 | 435 | 464 | 493 | 522 | 551 | 580 | . 4 | 12 |
| 95 | $-323$ | $-352$ | -381 | -410 | -440 | -469 | -498 | -528 | -557 | -586 |  | 15 |
| 96 | 326 | 356 | 385 | 415 | 444 | 474 | 504 | 533 | 563 | 593 | . 6 | 18 |
| 97 | 329 | 359 | 389 | 419 | 449 | 479 | 509 | 539 | 569 | 599 | . 7 | 21 |
| 98 | ${ }_{3} 33$ | 363 | ${ }^{393}$ | 423 | 454 | 484 | 514 | 544 | 575 | 605 | . 8 | ${ }_{2}^{23}$ |
| 99 | 336 | 367 | 397 | 428 | 458 | 489 | 519 | 550 | 581 | 611 |  | 26 |
| 100 | 540 | $3{ }^{2} 0$ | 401 | 432 | 463 | 494 | 525 | 556 | 586 | 617 |  |  |
|  | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |  |  |
|  | . 1 | 2 | $\cdot 3$ | . 4 | . 5 | . 6 | 7 | . 8 | $\cdot 9$ |  |  |  |
|  | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |  |  |  |

TABLE XI.-Continued.
Volumes by the Prismoidal Formula.

| n号0 | Heights. |  |  |  |  |  |  |  |  |  | Corrections for tenths in height. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |  |  |
| 1 | 6 | 7 | 7 | 7 | 8 | 8 | 8 | 9 | 9 | - | . 1 | 0 |
| 2 | 13 | 14 | 14 | 15 | 15 | 16 | 17 | 17 | 18 | 19 | . 2 | 0 |
| 3 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | $\cdot .3$ | 0 |
| 4 | 26 | 27 | 28 | 30 | 31 | 32 | 33 | 35 | 36 | 37 | . 4 | 1 |
| ${ }_{6}^{5}$ | -32 -39 | -34 41 | $\begin{array}{r}-35 \\ \hline 43\end{array}$ | $\begin{array}{r}-37 \\ \hline 4\end{array}$ | -39 46 | $\begin{array}{r}-40 \\ \hline 48\end{array}$ | -42 -50 | -43 | -45 -54 | -46 -56 | . .6 | 1 |
| 7 | 45 | 48 | 50 | 52 | 54 | 56 | 58 | 60 | 64 | 56 | . 6 | 1 |
| 8 | 52 | 54 | 57 | 59 | 62 | 64 | 67 | 69 | 72 | 76 | $\cdot 7$ | 1 |
| 9 | 58 | 61 | 64 | 67 | 69 | 72 | 75 | 78 | 81 | 83 |  | 1 |
| 10 | 65 | 68 | 71 | 74 | 77 | 80 | 83 | 86 | 90 | 93 |  | 1 |
| 11 | 71 | 75 | 78 | 81 | 85 | 88 | 92 | 95 | 98 | 102 | . 1 | 0 |
| 12 | 78 | 81 | 85 | 89 | 93 | 96 | 100 | 104 | 107 | 111 | . 2 | 1 |
| 13 | 84 | 88 | 92 | 96 | 100 | 114 | 108 | 112 | 116 | 120 | . 3 | 1 |
| 14 | 91 | 95 | 99 | 104 | 108 | 112 | 117 | 121 | 125 | 130 | . 4 | 2 |
| 15 | -97 | -102 | -106 | -111 | -116 | -120 | -125 | -130 | -134 | -139 |  | 2 |
| 16 | 104 | 109 | 114 | 119 | 123 | 128 | 133 | 138 | 143 | 148 | . 6 | 3 |
| 17 | 110 | 115 | 121 | 126 | 131 | 186 | 142 | 147 | 152 | 157 | . 7 | 3 |
| 18 | 117 | 122 | 128 | 133 | 139 | 144 | 150 | 156 | 161 | 167 | . 8 | 4 |
| 19 | 123 | 129 | 135 | 141 | 147 | 152 | 158 | 164 | 170 | 176 | . 9 | 4 |
| 20 | 130 | 136 | 142 | 148 | 154 | 160 | 167 | 173 | 179 | 185 |  |  |
| 21 | 136 | 142 | 149 | 156 | 162 | 169 | 175 | 181 | 188 | 194 | 1 | 1 |
| 22 | 143 | 149 | 156 | 163 | 170 | 177 | 183 | 190 | 197 | 204 | .2 | 2 |
| 23 | 149 | 156 | 163 | 170 | 177 | 185 | 192 | 199 | 206 | 213 | $\cdot 3$ |  |
| 24 | 156 | 163 | 170 | -178 | 185 -193 | 193 | -200 | - 207 | 215 | 222 | -4 | 3 |
| 25 | -162 | -170 $17 \%$ | -177 185 | -185 | -193 -201 | -201 -209 | -208 | -216 -225 | -224 -233 | -231 |  | 4 |
| ${ }_{27}^{26}$ | 169 175 | 177 183 | 185 192 | 193 | 201 | 209 | 217 225 | 225 | 233 242 | 241 250 | . 6 | 5 5 |
| 28 | 181 | 190 | 199 | 207 | 216 | 225 | 233 | 242 | 251 | 259 | . 8 | ${ }_{6}$ |
| 29 | 188 | 197 | 206 | 215 | $2 \% 4$ | 233 | R42 | 251 | 260 | 269 | . 9 | 7 |
| 30 | 194 | 204 | 213 | 22.2 | 231 | 241 | 250 | 259 | 269 | 278 |  |  |
| 31 | 201 | 210 | 220 | 230 | 239 | 249 | 258 | 268 | 277 | 287 | . 1 |  |
| 32 | 207 | 217 | 227 | 237 | 247 | 257 | 267 | 277 | 286 | 296 | . 2 | 2 |
| 33 | 214 | 224 | 234 | 244 | 255 | 265 | 275 | 285 | 295 | 306 | $\cdot 3$ | 3 |
| 34 | 220 | 231 | 241 | 25.2 | 262 | 273 | 283 | 294 | 304 | 315 | . 4 | 4 |
| 35 | -227 | -238 | -248 | -259 | -2\%0 | -281 | -292 | -302 | -313 | -324 |  | 5 |
| 36 | 233 | 244 | 256 | 267 | 278 | 289 | 300 | 311 | 322 | 333 | . 6 | 6 |
| 37 | 240 | 251 | 263 | 274 | 285 | 297 | 308 | 320 | 331 | 343 | $\cdot 7$ | 8 |
| 38 | 246 | 258 | 270 | 281 | 293 | 305 | 317 | 3328 | 340 | 352 | . 8 | 9 |
| 39 | 253 | 265 | 277 | 289 | 301 | 313 | 325 | 337 | 349 | 361 | . 9 | 10 |
| 40 | 259 | 272 | 284 | 296 | 309 | 321 | 333 | 346 | 358 | 370 |  |  |
| 41 | ${ }_{272}^{266}$ | ${ }_{285}^{278}$ | 291 | 304 311 | 316 324 | 329 337 | 342 350 | 354 363 | 367 376 | 380 389 | I | 3 |
| 43 | $2 \% 9$ | 292 | 2305 | 319 | ${ }_{3}^{324}$ | ${ }_{345}^{337}$ | 350 | 363 372 3 | 376 <br> 385 | 389 398 | . 2 | 3 4 |
| 44 | 285 | 299 | 312 | 326 | 340 | 353 | 367 | 380 | 394 | 407 | . 4 | 6 |
| 45 | -292 | $-306$ | -319 | $-333$ | $-347$ | -361 | -375 | -389 | $-403$ | -417 | . 5 | 7 |
| 46 | 298 | 312 | 327 | 341 | 355 | 369 | 383 | 398 | 412 | 426 | . 6 | 8 |
| 47 | 305 | 319 | 334 | 348 | 363 | 377 | 392 | 406 | 421 | 435 | . 7 | 10 |
| 48 | 311 | 326 | 341 | 356 | 370 | 385 | 400 | 415 | 430 | 444 | . 8 | 11 |
| 49 | 318 | 333 | 348 | 363 370 | 378 386 | 393 | 408 | 423 | 439 | 454 | . 9 | 13 |
| 50 | 324 | 340 | 355 | 370 | 386 | 401 | 417 | 432 | 448 | 463 |  |  |
|  | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |  |  |
|  | 1 | . 2 | $\cdot 3$ | 4 | . 5 | . 6 | 7 | . 8 | . 9 | Corrections for tenths in width. |  |  |
|  | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |  |  |  |  |  |

TABLE XI.-Continued.
Volumes by the Prismoidal Formula.

| $\begin{aligned} & \text { 雨 } \\ & \\ & \hline \end{aligned}$ | Heights. |  |  |  |  |  |  |  |  |  | Corrections for tenths in height. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |  |  |
| 51 | 331 | 346 | 362 | 378 | 394 | 409 | 425 | 441 | 456 | 472 | . 1 | 2 |
| 52 | 337 | 353 | 369 | 385 | 401 | 417 | 433 | 449 | 465 | 481 | .2 | 3 |
| 53 | 344 | 360 | 376 | 393 | 409 | 425 | 442 | 458 | 474 | 491 | $\cdot 3$ | 5 |
| 54 | 350 | 367 | 383 | 400 | 417 | 433 | 450 | 467 | 483 | 500 | . 4 | 7 |
| 55 | 356 | -373 | -390 | -407 | -424 | 441 | -458 | -475 | -492 | -509 | . 5 | 8 |
| 56 | 363 | 380 | 398 | 415 | 432 | 449 | 467 | 484 | 501 | 519 | . 6 | 10 |
| 57 | 369 | 387 | 405 | 422 | 440 | 457 | 475 | 493 | 510 | 528 | .7 | 12 |
| 58 | 376 | 394 | 412 | 430 | 448 | 465 | 483 | 501 | 519 | 537 | . 8 | 14 |
| 59 | 382 | 401 | 419 | 437 | 455 | 473 | 492 | 510 | 528 | 546 | . 9 | 15 |
| 60 | 389 | 407 | 426 | 444 | 463 | 481 | 500 | 519 | 537 | 556 |  |  |
| 61 | 395 | 414 | 433 | 452 | 471 | 490 | 508 | 527 | 546 | 565 | . 1 | 2 |
| 62 | 402 | 421 | 440 | 459 | 478 | 438 | 517 | 536 | 555 | 574 | . 2 | 4 |
| 63 | 408 | 428 | 447 | 467 | 486 | 506 | 525 | 544 | 564 | 583 | $\cdot 3$ | 6 |
| 64 | 415 | 435 | 454 | 474 | 494 | 514 | 533 | 553 | 573 | 593 | - 4 | 8 |
| 65 | -421 | $-441$ | -461 | -481 | -502 | -522 | -542 | $-562$ | -582 | -602 | . 5 | 10 |
| 66 | 428 | 448 | 469 | 489 | 509 | 530 | 550 | 570 | 591 | 611 | . 6 | 12 |
| 67 | 434 | 455 | 476 | 496 | 517 | 538 | 558 | 579 | 600 | 620 | $\cdot 7$ | 14 |
| 68 | 441 | 462 | 483 | 504 | 525 | 546 | 567 | 588 | 609 | 630 | . 8 | 16 |
| 69 | 447 | 469 | 490 | 511 | 532 | 554 | 575 | 596 | 618 | 639 | . 9 | 18 |
| 70 | 454 | 475 | 497 | 519 | 540 | 562 | 583 | 605 | 627 | 648 |  |  |
| 71 | 460 | 482 | 504 | 526 | 548 | 570 | 592 | 614 | 635 | 657 | 1 | 2 |
| 72 | 467 | 489 | 511 | 533 | 556 | 578 | 600 | 622 | 644 | 667 | . 2 | 5 |
| 73 | 473 | 496 | 518 | 541 | 563 | 586 | 608 | 631 | 653 | 676 | $\cdot 3$ | 7 |
| 74 | 480 | 502 | 525 | 548 | 571 | 594 | 617 | 640 | 662 | 685 | - 4 | 9 |
| 75 | -486 | -509 | -532 | -556 | -579 | -601 | -625 | -648 | -671 | -694 | . 5 | 12 |
| 76 | 493 | 516 | 540 | 563 | 586 | 610 | 633 | 657 | 680 | 704 | . 6 | 14 |
| 78 | 499 | 523 | 547 | 570 | 594 | 618 | 642 | 665 | 689 | 713 | . 7 | 16 |
| 78 | 506 | 530 | 554 | 578 | 602 | 626 | 650 | 674 | 698 | 722 | . 8 | 19 |
| 79 | 512 | 536 | 561 | 585 | 610 | 634 | 658 | 683 | 707 | 731 | . 9 | 21 |
| 80 | 519 | 543 | 568 | 593 | 617 | 642 | 667 | 691 | 716 | 741 |  |  |
| 81 | 525 | 550 | 575 | 600 | 625 | 650 | 675 | 700 | 725 | 750 | . 1 | 3 |
| 82 | 531 | 557 | 582 | 607 | 633 | 658 | 683 | 709 | 734 | 759 | . 2 | 8 |
| 83 | 538 | 564 | 589 | 615 | 640 | 666 | 692 | 717 | 743 | 769 | $\cdot 3$ | 8 |
| 84 | 544 | 570 | 596 | $6: 2$ | 648 | 674 | 700 | 726 | 752 | 778 | - 4 | 10 |
| 85 | $-551$ | -577 | -603 | -630 | -656 | -682 | -708 | -735 | -761 | -787 | . 5 | 13 |
| 86 | 557 | 584 | 610 | 637 | 664 | 690 | 717 | 743 | 770 | 796 | . 6 | 16 |
| 87 | 564 | 591 | 618 | 644 | 671 | 698 | 725 | 752 | 779 | 806 | $\cdot 7$ | 18 |
| 88 | 570 | 598 | 625 | 652 | 679 | 706 | 733 | 760 | 788 | 815 | . 8 | 21 |
| 89 | 577 | 604 | 632 | 659 | 687 | 714 | 742 | 769 | 797 | 824 | . 9 | 24 |
| 90 | 583 | 611 | 639 | 667 | 694 | 722 | 750 | 777 | 806 | 833 |  |  |
| 91 | 590 | 618 | 646 | 674 | 702 | 730 | 758 | 786 | 815 | 843 | I |  |
| 92 | 596 | 625 | 653 | 681 | 710 | 738 | 767 | 795 | 823 | 852 | . 2 | 6 |
| 93 | 603 | 631 | 660 | 689 | 718 | 746 | 775 | 804 | 832 | 861 | $\cdot 3$ | 9 |
| 94 | 609 | 638 | 667 | 696 |  | 754 | 783 | 812 | -841 | 870 | - 4 | 12 |
| 95 | -616 | -645 | -674 | -704 | -733 | -762 | -792 | -821 | -850 | -880 | . 5 | 15 |
| 96 | 622 | 652 | 681 | 711 | 741 | 770 | 800 | 830 | 859 | 889 | . 6 | 18 |
| 97 | 629 | 659 | 689 | 719 | 748 | 778 | 808 | 838 | 868 | 898 | . 7 | $\stackrel{21}{21}$ |
| 98 | 435 | 665 | 696 | 720 | 756 | 786 | 817 | 847 | 877 | 907 | . 8 | 23 |
| 99 | 642 | 672 | 703 | 733 | 764 | 794 | 825 | 856 | 886 | 917 | . 9 | 26 |
| 100 | 648 | 679 | 710 | 741 | 772 | 802 | 833 | 864 | 895 | 926 |  |  |
|  | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |  |  |
|  | . 1 | . 2 | $\cdot 3$ | . 4 | $\cdot 5$ | . 6 | . 7 | . 8 | . 9 | Corrections for tenths in width. |  |  |
|  | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |  |  |  |  |  |

TABLE XI.-Continued.
Volumes by the Prismoidal Formula.

| $\stackrel{\text { ¢ }}{\text { ¢ }}$ |  |  |  |  |  | ghts. |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |  |  |
|  | 10 | 10 | 10 | 10 | 11 |  |  |  |  |  |  |  |
| $\overline{3}$ | 19 29 | 30 | ${ }_{31}^{20}$ | 21 31 | ${ }_{3}^{2 \cdot}$ | 22 <br> 3 <br> 3 | 123 34 | ${ }^{23}$ | 24 | 25 | . 2 | 0 |
| 4 | 38 | 40 | 41 | 42 | ${ }_{43}$ | ${ }_{44}$ | ${ }_{46} 44$ | $\begin{aligned} & 35 \\ & 47 \end{aligned}$ | 36 48 | 37 49 49 | ${ }^{4} 4$ | 1 |
| 5 | -48 | -49 | $-51$ | - 58 | -54 | - ${ }^{44}$ | - ${ }^{46}$ | -59 | -60 | - ${ }^{49}$ | .4 | 1 |
| ${ }_{7}$ | ${ }^{57}$ | 59 | ${ }_{51}^{61}$ | ${ }_{-3}^{63}$ | ${ }^{65}$ | ${ }^{67}$ | 68 | 70 | 72 | ${ }^{7}$ | . 6 | 1 |
| 8 | ${ }_{77}^{67}$ | $\stackrel{69}{79}$ | ${ }_{81}^{71}$ | 73 84 84 | ${ }^{76}$ | 78 89 | 80 91 | 82 | 84 96 | ${ }_{97}^{86}$ | . 8 | 1 |
| 9 | 86 | 89 | 92 | 94 | 97 | 100 | 103 | 106 | 108 | 111 | . 9 | 1 |
| 10 | 96 | 99 | 102 | 105 | 108 | 111 | 114 | 117 | 120 | 123 |  |  |
| 11 | 105 | 109 | 112 | 115 | 119 | 122 | 126 | 129 | 132 | 136 | . 1 |  |
| ${ }_{13}^{12}$ | 115 | 119 128 | ${ }_{13,2}^{123}$ | 126 136 | 130 <br> 140 <br> 1 | 133 <br> 144 <br> 1 | 137 | 141 | 114 | 148 | 2 | 1 |
| 14 | 134 | 138 | ${ }_{143}^{133}$ | ${ }_{14 \sim}^{136}$ | 151 | 144 156 | 148 | 152 164 168 | 156 169 | 160 173 | $\stackrel{3}{4}$ | ${ }_{2}^{1}$ |
| 15 | -144 | -148 | -153 | -157 | -162 | -167 | -171 | -176 | -181 | -185 | - 5 | 2 |
| 16 | ${ }^{153}$ | 158 | 163 | 168 | 173 | 178 | 183 | 188 | 193 | 198 | . 6 | 3 |
| 18 | ${ }_{173}^{163}$ | 168 | 173 | 178 | 18.3 | 189 | ${ }^{194}$ | 199 | 205 | 210 | . 7 | 3 |
| . 18 | 182 | 1188 188 | 194 | 189 199 | ${ }_{205}^{194}$ | 211 | ${ }_{217}^{206}$ | 211 223 | 217 29 | ${ }_{235}^{222}$ | . 8 | 4 |
| 20 | 191 | 198 | 204 | 210 | 216 | 222 | 238 | 235 | 241 | 247 | . 9 |  |
| 21 | 201 | 207 | 214 | 220 | 227 | 233 | 240 | 246 | 253 | 259 | I |  |
| ${ }_{23}^{22}$ | 210 220 | ${ }_{2}^{217}$ | \% 24 | ${ }_{241}^{231}$ | ${ }_{248}^{238}$ | $\stackrel{24}{246}$ | ${ }_{263}^{251}$ | ${ }_{2}^{258}$ | ${ }_{2}^{265}$ | 272 | . 2 | ${ }_{2}^{2}$ |
| 24 | 230 | 237 | 244 | $\stackrel{253}{24}$ | $\stackrel{2}{259}$ | ${ }_{267}^{256}$ | 274 | ${ }_{281}^{280}$ | $\stackrel{287}{289}$ | ${ }_{296}^{284}$ | . 3 | ${ }_{3}^{2}$ |
| 25 | 239 | $24 \%$ | 255 | $26^{3}$ |  |  | 28 |  | ${ }^{301}$ | 309 |  |  |
| ${ }_{2}^{26}$ | $\stackrel{249}{298}$ | ${ }_{6}^{257}$ | 265 | ${ }_{28}^{273}$ | 281 | 289 | $\stackrel{297}{ }$ | ${ }_{3}^{305}$ | ${ }^{313}$ | 321 | . 6 | 5 |
| 28 | 268 | $2{ }^{2} 7$ | 285 | 294 | 302 | 311 | ${ }_{3 * 0}^{308}$ | 328 | ${ }_{337}$ | ${ }_{346}$ | . 8 | ${ }_{6}^{5}$ |
| 29 | ${ }_{287}^{277}$ | 286 | ${ }_{3}^{295}$ | 304 | 313 | 3\% | ${ }^{331}$ | 340 | 349 | 3588 | . 9 | 7 |
| 30 | 287 | 296 | 306 | 315 | 3\%t | 333 | 343 | 352 | 361 | 370 |  |  |
| 31 | 297 | 306 | 316 | 325 | 335 | 344 | 354 | 364 | 373 | 383 | . 1 |  |
| ${ }_{33}$ | 316 | ${ }_{326}$ | 326 336 3 | -336 | 346 <br> 356 | - | ${ }^{365}$ | ${ }_{387}^{375}$ | - | ${ }^{395}$ | . 2 | $\stackrel{2}{3}$ |
| 34 | 3:5 | 336 | 346 | 357 | 367 | $3{ }^{3} 8$ | 388 | 399 | 409 | 420 | . 4 | 4 |
| 35 | 335 | 346 | 356 | 367 | 378 | 389 | 400 | 410 | 421 | 432 |  | 5 |
| 36 | 344 | ${ }^{356}$ | ${ }^{367}$ | ${ }^{378}$ | ${ }^{389}$ | 400 | 411 | 422 | 433 | 444 | . 6 | ${ }_{8}^{6}$ |
| ${ }_{38}^{37}$ | 354 <br> 364 | 365 375 | 377 <br> 387 | 388 399 |  | ${ }_{422}^{411}$ | 434 | ${ }_{446}^{434}$ | $4{ }_{4}^{457}$ | ${ }_{469}$ | . 78 | ${ }_{9}^{8}$ |
| 39 | 373 | 385 | 897 | 409 | 421 | 433 | 445 | 457 | 469 | 481 | . 9 | 10 |
| 40 | 383 | 395 | 407 | 420 | 432 | 444 | 457 | 469 | 481 | 494 |  |  |
| 41 | 392 | 405 | 418 | 430 | 443 | 456 | 468 | 481 | 494 | 506 |  |  |
| 43 | 411 | $4{ }_{4}^{45}$ | ${ }_{438}$ | 441 | 454 | 467 478 | 489 | 493 504 | 506 518 | 519 | . 2 | 3 4 4 |
| 44 | 421 | 435 | 448 | 462 | 475 | 489 | 502 | 516 | 530 | 543 | ${ }^{-3}$ |  |
| 45 | 431 | 444 | 458 | 472 | 487 | -500 | -514 | 528 | 542 | -556 | - 5 | 7 |
| 46 | 440 | 454 | ${ }_{4}^{469}$ | 483 | 497 | 511 | 525 | 540 | 554 | 568 | . 6 | 8 |
| 48 | 459 | ${ }_{474}^{464}$ | ${ }_{489}$ | 493 | 508 519 | 522 | 537 | 551 563 | 年 566 | 580 593 | . 78 | 10 |
| 49 | 469 | 484 | 499 | 514 | 529 | 544 | 560 | 575 | 590 | 605 |  |  |
| 50 | 478 | 491 | 509 | 525 | 540 | 556 | 571 | 586 | 60.2 | 617 |  |  |
|  | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |  |  |
|  | . 1 | . 2 | $\cdot 3$ | $\cdot 4$ | . 5 | . 6 | . 7 | . 8 | -9 | Corrections for tenths in width. |  |  |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 |  |  |  |

TABLE XI.-Continued.
Volumes by the Prismoidal Formula.

|  | Heights. |  |  |  |  |  |  |  |  |  | Corrections for tenths in height. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |  |  |
| 51 | 488 | 504 | 519 | 535 | 551 | 567 | 582 | 598 | 614 | 630 | . 1 | 2 |
| 52 | 498 | 514 | 530 | 546 | 562 | 5 58 | 594 | 610 | 626 | 642 | .2 | 3 |
| 53 | 507 | 523 | 540 | 556 | 573 | 589 | 605 | 622 | 638 | 654 | $\cdot 3$ | 5 |
| 54 | 517 | 533 | 550 | 567 | 583 | 600 | 617 | 633 | 650 | 667 | . 4 | 7 |
| 55 | - 526 | $-543$ | -560 | $-577$ | $-594$ | -611 | -628 | -645 | $-662$ | -679 | - 5 | 8 |
| 56 | 536 | 553 | $5{ }_{5} 0$ | 588 | 605 | 623 | 640 | 657 | 674 | 691 | . 6 | 10 |
| 57 | 545 | 563 | 581 | 598 | 616 | 633 | 651 | 669 | 686 | 704 | . 7 | 12 |
| 58 | 555 | 573 | 591 | 609 | 627 | 644 | 662 | 680 | 698 | 716 | . 8 | 14 |
| 59 | 565 | 583 | 601 | 619 | 637 | 656 | 674 | 692 | 710 | 728 | . 9 | 15 |
| 60 | 574 | 593 | 611 | 630 | 648 | 667 | 685 | 704 | 722 | 741 |  |  |
| 61 | 584 | 602 | 621 | 640 | 659 | 678 | 697 | 715 | 734 | 753 | . 1 | 2 |
| 62 | 593 | 612 | 631 | 651 | 670 | 689 | ro8 | 727 | 746 | 765 | . 2 | 4 |
| 63 | 603 | 622 | 642 | 661 | 681 | 700 | 719 | 739 | 758 | 778 | $\cdot 3$ | 6 |
| 64 | 612 | 632 | 652 | 672 | 691 | 711 | 731 | 751 | 770 | 790 | . 4 | 8 |
| 65 | -612 | -642 | -662 | -682 | -702 | -722 | $-742$ | -762 | -782 | -802 | . 5 | 10 |
| 66 | 631 | 652 | $6{ }^{2} 2$ | 693 | 713 | 733 | 754 | 74 | 794 | 815 | . 6 | 12 |
| 67 | 641 | 662 | 632 | 703 | 774 | 744 | 765 | 786 | 806 | 827 | .7 | 14 |
| 68 | 651 | $6 \pi 2$ | 693 | 714 | 735 | 756 | 777 | 798 | 819 | 840 | . 8 | 16 |
| 69 | 660 | 681 | ${ }_{7} 7$ | 724 | 745 | 767 | 788 | 809 | 831 | 85.2 | .9 | 18 |
| 70 | $6{ }^{0} 0$ | 691 | 713 | 735 | 756 | $7 \% 8$ | 799 | 821 | 843 | 864 |  |  |
| 71 | 679 | 901 | 723 | 745 | 767 | 789 | 811 | 833 | 855 | 877 | . 1 | 2 |
| 72 | 689 | 711 | 733 | 756 | 778 | 800 | 822 | 844 | 867 | 889 | . 2 | 5 |
| 73 | 698 | 721 | 744 | 766 | 789 | 811 | 834 | 856 | 879 | 901 | $\cdot 3$ | 7 |
| 74 | 708 | $73 i$ | 754 | 777 | 799 | 822 | 845 | 868 | 891 | 914 | . 4 | 9 |
| 75 | - 718 | -741 | -764 | - 787 | -810 | -833 | -856 | -880 | $-903$ | -926 | . 5 | 12 |
| 76 | 727 | 751 | \%ri4 | 798 | 821 | 844 | 868 | 891 | 915 | 938 | . 6 | 14 |
| 77 | 737 | 760 | 784 | 808 | 832 | 856 | 879 | 903 | 927 | 951 | . 7 | 16 |
| 78 | 746 | 770 | 794 | 819 | 843 | 867 | 891 | 915 | 939 | 963 | . 8 | 19 |
| 79 | 756 | \%80 | 805 | 829 | 853 | 878 | 902 | 927 | 951 | 975 | . 9 | 21 |
| 80 | 765 | 790 | 815 | 840 | 864 | 889 | 914 | 938 | 963 | 988 |  |  |
| 81 | \%75 | 800 | 825 | 850 | 875 | 900 | 925 | 950 | 975 | 1000 | . 1 | 3 |
| 82 | 785 | 810 | 835 | 860 | 886 | 911 | 936 | 962 | 987 | 1012 | . 2 | 5 |
| 83 | 794 | 820 | 845 | 871 | 897 | 922 | 948 | 973 | 999 | 1025 | . 3 | 8 |
| 84 | 804 | 830 | 856 | 881 | 907 | 933 | 959 | 985 | 1011 | 1037 | . 4 | 10 |
| 85 | -813 | -840 | -866 | -892 | -918 | -944 | -971 | -99\% | -1023 | -1049 | . 5 | 13 |
| 86 | 823 | 849 | 876 | 902 | 929 | 956 | 982 | 1009 | 1035 | 1062 | . 6 | 16 |
| 87 | 832 | 859 | 886 | 913 | 940 | 967 | 994 | 1020 | 1047 | 1074 | . 7 | 18 |
| 88 | 842 | 869 | 896 | 923 | 951 | 978 | 1005 | 1032 | 1059 | 1086 | . 8 | 21 |
| 89 | 852 | 879 | 906 | 934 | 961 | 989 | 1016 | 1044 | 1071 | 1098 | . 9 | 24 |
| 90 | 861 | 889 | 917 | 944 | 972 | 1000 | 1028 | 1056 | 1083 | 1111 |  |  |
| 91 | 871 | 899 | 927 | 955 | 983 | 1011 | 1039 | 1067 | 1095 | 1123 | . 1 | 3 |
| 92 | 880 | 909 | 937 | 965 | 994 | 1022 | 1051 | 1079 | 1107 | 1136 | . 2 | 6 |
| 93 | 890 | 919 | 947 | 976 | 1005 | 1033 | 1062 | 1091 | 1119 | 1148 | . 3 | 9 |
| 94 | 899 | 928 | 957 | 986 | 1015 | 1044 | 1073 | 1102 | 1131 | 1160 | . 4 | 12 |
| 95 | $\begin{array}{r}-909 \\ \hline 919\end{array}$ | -938 | -968 | -997 | -1026 | -1056 | -1085 | -1114 | -1144 | -1173 | . 5 | 15 |
| 96 | 919 | 948 | 978 | 1007 | 1037 | 106\% | 1096 | 1126 | 1156 | 1185 | . 6 | 18 |
| 97 | 928 | 958 | 988 | 1018 | 1048 | 1078 | 1108 | 1138 | 1168 | 1198 | . 7 | 21 |
| 98 | 938 | 968 | 998 | 1028 | 1059 | 1089 | 1119 | 1149 | 1180 | 1210 | . 8 | 23 |
| 99 100 | 947 | 978 | 1008 | 1039 | 1069 | 1100 | 1131 | 1161 | 1192 | 1222 | $\cdot 9$ | 26 |
|  |  |  |  | 1049 | 108 | 111 | 142 | 1173 | 1204 | 1205 |  |  |
|  | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |  |  |
|  | . 1 | 2 | $\cdot 3$ | 4 | . 5 | . 6 | . 7 | . 8 | . 9 | Corrections for tenths in width. |  |  |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 |  |  |  |  |  |

TABLE XI.-Continued.
Volumes by the Prismoidal Formula.


TABLE XI.-Continued.
Volumes by the Prismoidal Formula.

| $\begin{aligned} & \text { 年 } \\ & 0 \\ & 0 \end{aligned}$ | Heights. |  |  |  |  |  |  |  |  |  | Corrections for tenths in height. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |  |  |
| 51 | 645 | 661 | 677 | 693 | 708 | 724 | 740 | 756 | 771 | 787 | . 1 | 2 |
| 52 | 658 | 674 | 690 | 806 | 722 | 738 | 754 | 770 | 786 | 802 | . 2 | 8 |
| 53 | 671 | 687 | 703 | 720 | 736 | 752 | 768 | 785 | 802 | 818 | . 3 | 5 |
| 54 | 683 | 700 | 717 | 733 | 750 | 767 | 783 | 800 | 817 | 833 | . 4 | 7 |
| 55 | -696 | $-713$ | -730 | -747 | -764 | -781 | -798 | -815 | -832 | -849 | - 5 |  |
| 56 | 709 | 726 | 743 | 760 | 778 | 795 | 812 | 830 | 847 | 864 | . 6 | 10 |
| 57 | 721 | 739 | 756 | 774 | 792 | 809 | 827 | 844 | 862 | 880 | .7 | 12 |
| 58 | 734 | 752 | 770 | 788 | 806 | 823 | 841 | 859 | 877 | 895 | . 8 | 14 |
| 59 | 747 | 765 | 783 | 801 | 819 | 833 | 856 | 874 | 892 | 910 | .9 | 15 |
| 60 | 759 | 778 | 796 | 815 | 833 | 852 | 870 | 889 | 907 | 926 |  |  |
| 61 | 772 | 791 | 810 | 828 | 847 | 866 | 885 | 994 | 923 | 941 | . 1 | 2 |
| 62 | \%85 | 804 | 823 | 842 | 861 | 880 | 899 | 919 | 938 | 957 | .2 | 4 |
| 63 | 797 | 817 | 836 | 856 | 875 | 894 | 914 | 933 | 953 | 972 | . 3 | 6 |
| 64 | 810 | 830 | 849 | 869 | 889 | 909 | 928 | 948 | 968 | 988 | . 4 | 8 |
| 65 | -823 | -843 | -863 | -883 | -903 | -923 | -943 | -963 | -983 | -1003 | . 5 | 10 |
| 66 | 835 | 856 | 876 | 896 | 917 | 937 | 957 | 978 | 998 | 1019 | . 6 | 12 |
| 67 | 848 | 869 | 889 | 910 | 931 | 951 | 972 | 993 | 1013 | 1034 | . 7 | 14 |
| 68 | 860 | 881 | 902 | 923 | 944 | 965 | 986 | 1007 | 1028 | 1049 | . 8 | 16 |
| 69 | 873 | 894 | 916 | 937 | 958 | 980 | 1001 | 1022 | 1044 | 1065 | .9 | 18 |
| 70 | 886 | 907 | 929 | 951 | 972 | 994 | 1015 | 1037 | 1059 | 1080 |  |  |
| 81 | 898 | 920 | 942 | 964 | 986 | 1008 | 1030 | 1052 | 1074 | 1096 | 1 | 2 |
| 72 | 911 | 933 | 956 | 978 | 1000 | 1022 | 1044 | 1067 | 1089 | 1111 | . 2 | 3 |
| 73 | 924 | 946 | 969 | 991 | 1014 | 1036 | 1059 | 1081 | 1104 | 1127 | $\cdot 3$ | 7 |
| 74 | 936 | 959 | -982 | 1005 | 1028 | 1051 | 1073 | 1096 | 1119 | 1142 | . 4 | 1 |
| 55 | -949 | $-972$ | -995 | -1019 | -1042 | -1065 | -1088 | -1111 | -1134 | $-1157$ | . 5 | 12 |
| 76 | 962 | 985 | 1009 | 1032 | 1056 | 1079 | 1102 | 1126 | 1149 | 1173 | . 6 | 14 |
| 77 | 974 | 998 | 1022 | 1046 | 1069 | 1093 | 117 | 1141 | 1165 | 1188 | . 7 | 16 |
| 78 | 987 | 1011 | 1035 | 1059 | 1083 | 1107 | 1131 | 1156 | 1180 | 1204 | . 8 | 19 |
| 79 | 1000 | 1024 | 1048 | 1073 | 1097 | 1122 | 1146 | 1170 | 1195 | 1219 | . 9 | 21 |
| 80 | 1012 | 1037 | 1062 | 1086 | 1111 | 1136 | 1160 | 1185 | 1210 | 1235 |  |  |
| 81 | 1025 | 1050 | 1075 | 1100 | 1125 | 1150 | 1175 | 1200 | 1225 | 1250 | . 1 | 8 |
| 82 | 1038 | 1063 | 1088 | 1114 | 1139 | 1164 | 1190 | 1215 | 1240 | 1265 | .2 | 8 |
| 83 | 1050 | 1076 | 1102 | 1127 | 1153 | 1178 | 1204 | 1230 | 1255 | 1281 | $\cdot 3$ | 8 |
| 84 | 1063 | 1089 | 1115 | 1141 | 1167 | 1193 | 1219 | 1244 | 1270 | 1296 | . 4 | 10 |
| 85 | -1076 | -1102 | -1128 | -1154 | -1181 | -1207 | -1233 | -1259 | -1285 | $-1312$ |  | 13 |
| 86 | 1088 | 1115 | 1141 | 1168 | 1194 | 1221 | 1248 | 1274 | 1301 | 1327 | . 6 | 16 |
| 87 | 1101 | 1128 | 1155 | 1181 | 1208 | 1235 | 1262. | 1289 | 1316 | 1343 | $\cdot 7$ | 18 |
| 88 | 1114 | 1141 | 1168 | 1195 | 1222 | 1249 | $127 \%$ | 1304 | 1331 | 1358 | . 8 | 21 |
| 89 | 1126 | 1154 | 1181 | 1209 | 1236 | 1264 | 1291 | 1319 | 1346 | 1373 | . 9 | 24 |
| 90 | 1139 | 1167 | 1194 | 1222 | 1250 | 1278 | 1306 | 1333 | 1361 | 1389 |  |  |
| 91 | 1152 | 1180 | 1208 | 1236 | 1264 | 1292 | 1320 | 1348 | 1376 | 1404 | . 1 | 3 |
| 92 | 1164 | 1193 | 1221 | 1249 | 1278 | 1306 | 1335 | 1363 | 1391 | 1420 | . 2 | 6 |
| 93 | 1177 | 1206 | 1234 | 1263 | 1292 | 1320 | 1349 | 1378 | 1406 | 1435 | $\cdot 3$ | 9 |
| 94 | 1190 | 1219 | 1248 | 1277 | 1306 | 1335 | 1364 | 1393 | 1422 | 1451 | . 4 | 12 |
| 95 | -1202 | -1231 | -1261 | -1290 | -1319 | -1349 | -1378 | -1407 | -1437 | -1466 | . 5 | 15 |
| 96 | 1215 | 1244 | 1274 | 1304 | 1333 | 1363 | 1393 | 1422 | 1452 | 1481 | . 6 | 18 |
| 97 | 1227 | 1257 | 1287 | 1317 | 1347 | 1377 | 1407 | 1437 | 1467 | 1497 |  | 21 |
| 98 | 1240 | 1270 | 1301 | 1331 | 1361 | 1391 | 1422 | 1452 | 1482 | 1512 | . 8 | 23 |
| 99 | 1253 | 1283 | 1314 | 1344 | 1375 | 1406 | 1436 | 1467 | 1497 | 1528 | . 9 | 26 |
| 100 | 1265 | 1296 | 1327 | 1358 | 1389 | 1420 | 1451 | 1481 | 1512 | 1543 |  |  |
|  | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |  |  |
|  | . 1 | 2 | -3 | 4 | $\cdot 5$ | . 6 | . 7 | . 8 | -9 | Corrections for tenths in width. |  |  |
|  | 1 | 3 | 4 | 6 | 7 | 8 | 10 | 11 | 13 |  |  |  |  |  |

The Star and the Azimuth are W. of N. when the hour angle is less
The Argument is the star's hour angle (or 23 h . 56 min .
To Find tee True Meridian the azimuth must be laid off to the east when the

for all Hour Angles. § 381a.
than $11^{\mathrm{h}} 58^{\mathrm{m}}$ and E . of N. when the hour angle is greater than $11^{\mathrm{b}} 58^{\mathrm{m}}$. minus the star's hour angle), for the years given.
hour angle is less than $11^{\mathrm{h}} 58 \mathrm{~m}$, and to the west when it is greater than $11 \mathrm{~h} 58^{\mathrm{m}}$.


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RENERENCE


[^0]:    * This is shown in mechanics, and the student may have to take it for granted temporarily.

[^1]:    * For the demonstration of the prismoidal formula see Art. 314.

[^2]:    * This figure is taken from a paper describing the method by Prof. William G. Raymond, University of California.

[^3]:    * For a demonstration of this formula see Henck's Field-Book.

[^4]:    * Tables for Computing the Cubic Contents of Excavations and Embankments. By John R. Hudson, C.E. John Wiley \& Sons, New York, 1884.

[^5]:    * The two methods here discussed are the only ones that have any claims to accuracy. The method by "mean end areas," wherein the volume is assumed to be the mean of the end areas into the length, always gives too great a volume (except when a greater centre height is found in connection with a less total width, which seldom occurs), the excess being one sixth of the volume of the pyramids involved in the elementary forms of the prismoid. This is a large error even in level sections, and very much greater on sloping ground, and yet it is the basis of most of the tables used in computing earthwork, and in some States it is legalized by statute. Thus in the example computed by Henck's method on p. 414 the volume by mean end areas is 1193 cu . yards; by the prismoidal formula it is 1168 cu . yards, while by the method by diagonals it was only 1001 cu . yards. This was an extreme case, however, and was selected to show the adaptation of the method by diagonals to such a form.

[^6]:    * For a further exposition of this subject, see Appendix C.

[^7]:    * See paper by P. J. Flynn in Trans. Tech. Soc. of the Pacific Coast, voL ii. p. 179, where all the available experimental data are given.

[^8]:    * For the derivation of this rule see Appendix C.

[^9]:    *This table was computed by Mr. Arthur Winslow of the State Geological Survey of Pennsylvania.
    For description of chart for graphical reduction see p. v.

